

Instructions

- Introduce yourselves! Despite popular belief, math is in fact a team sport!
- Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
- Do the problems below, having a different person be the scribe for each one.
- Try to work out the problems as a group, but feel free to flag me down if you run into a wall.

Proofs!

1. Prove that if $3n + 2$ is odd, then n is odd.
2. Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.
3. Prove that there is a pair of consecutive integers such that one is a perfect square and the other is a perfect cube.
4. What's wrong with the following "proof" that if $m^2 = n^2$, then $m = n$?
"We want to show $m = n$. Squaring both sides gives $m^2 = n^2$, which is true by hypothesis. Therefore, we must have $m = n$."
5. Prove that the sum of a rational number and an irrational number is irrational.
6. Prove that there are irrational numbers x, y such that x^y is rational
7. Prove that if a, b, n are positive integers such that $ab > n$, then $a > \sqrt{n}$ or $b > \sqrt{n}$
8. Prove that there are no solutions in integers to $2x^2 + 5y^2 = 14$
9. Show that you cannot tile a 10×10 board with 1×4 rectangular pieces
10. Suppose you have a big bar of chocolate that is $m \times n$ pieces large (think Hershey bar) and you want to break it up into its mn pieces. At each step, you can pick up any piece you currently have and break it along any line (again, think breaking up a Hershey bar) but you cannot "stack" two pieces on top of each other and break them together. Prove that no matter what sequence of breaks you make, it **always** takes $mn - 1$ steps to fully break down the chocolate bar.

Logic Puzzles

1. Consider the following set of four statements:
 - (a) One of these statements is false
 - (b) Two of these are false
 - (c) Three of these are false
 - (d) Four of these are false

Which of the above, if any, are true?

2. Four people travelling at night come to a small footbridge and need to cross to the other side. Unfortunately, they only have 1 flashlight and the bridge can only hold the weight of two people at once. One person takes 10 minutes to cross, another 5, another 2, and the last takes 1 minute. Anyone crossing must have the flashlight and when travelling together, they must go the pace of the slower person. Can they all get safely across in 17 minutes?