

Instructions

- Introduce yourselves! Despite popular belief, math is in fact a team sport!
- Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
- Do the problems below, having a different person be the scribe for each one.
- Try to work out the problems as a group, but feel free to flag me down if you run into a wall.

Rules of Inference

1. Determine what conclusions, if any, you can draw from each of the following sets of premises:
 - (a) “All athletes watch professional sports,” “Alice is an athlete”
 - (b) “All athletes watch professional sports,” “Bob watches professional sports”
 - (c) “All athletes watch pro sports,” “Everyone who watches sports likes pizza,” “Charlie doesn’t like pizza”
 - (d) “All athletes watch sports,” “Dan isn’t an athlete”
2. What’s wrong with each of the following arguments?
 - (a) Every CS major takes discrete math. Alice is taking discrete math. Therefore, Alice is a CS major.
 - (b) Every CS major takes discrete math. Bob is not a CS major. Therefore, Bob doesn’t take discrete math.
 - (c) If $n > 1$, then $n^2 > 1$. Since $(-5)^2 = 25 > 1$, $-5 > 1$

Basic Proof Methodology

1. Prove that if $3n + 2$ is odd, then n is odd.
2. Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.
3. What’s wrong with the following “proof” that if $m^2 = n^2$, then $m = n$?

“We want to show $m = n$. Squaring both sides gives $m^2 = n^2$, which is true by hypothesis. Therefore, we must have $m = n$.”
4. Prove that the sum of a rational number and an irrational number is irrational.
5. Prove that if a, b, n are positive integers such that $ab > n$, then $a > \sqrt{n}$ or $b > \sqrt{n}$

Logic Puzzles

1. For their final exam, an evil math 55 professor has put a small mark on the forehead of 55 of his 555 students and will fail the entire class unless they can determine which of them has the mark. The students are not allowed to talk or communicate with each other in any way nor do they know how many of them were marked, but they can see each others foreheads. Once per minute, the Professor asks for all the marked students to come to the front of the room. If any unmarked students come, or if more than 1 hour passes, he will fail the entire class. At what minute do the 55 marked students turn themselves in?
2. Consider the following set of four statements:
 - (a) One of these statements is false
 - (b) Two of these are false
 - (c) Three of these are false
 - (d) Four of these are false

Which of the above, if any, are true?

3. (Not really logic per-se, but a good thinking exercise) Two bicyclists enter opposite ends of a 100-foot long tunnel that is only wide enough for one bike. One is travelling 10 ft/s and the other is travelling 5 ft/s in the opposite direction. A bird flying 20 ft/s enters the tunnel just in front of the 10 ft/s cyclist. When the bird gets to the other cyclist, it immediately turns around and flies back towards the 10 ft/s cyclist. When the bird gets back to him, it turns around again, etc, etc. How many feet does the bird fly before the bicyclists collide?
4. Four people travelling at night come to a small footbridge and need to cross to the other side. Unfortunately, they only have 1 flashlight and the bridge can only hold the weight of two people at once. One person takes 10 minutes to cross, another 5, another 2, and the last takes 1 minute. Anyone crossing must have the flashlight and when travelling together, they must go the pace of the slower person. Can they all get safely across in 17 minutes?