

Instructions

- Introduce yourselves! Despite popular belief, math is in fact a team sport!
- Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
- Do the problems below, having a different person be the scribe for each one.
- Try to work out the problems as a group, but feel free to flag me down if you run into a wall.

Logic Symbols and Truth Tables

1. Let c = "It is cloudy," r = "It is raining", s = "It is sunny", w = "The ground is wet", n = "It is night". Translate each of the following into English

- (a) $c \rightarrow \neg s$
 (b) $\neg s \rightarrow c \vee n$
 (c) $r \rightarrow w \wedge c$

2. Using the same propositional variables as above, translate each of the following into formal logical statements:

- (a) It is raining whenever the ground is wet.
 (b) If it's sunny, then it's not raining.
 (c) It only rains at night.

3. Create a truth table for each of the following expressions. Determine whether each is a tautology, a contradiction, or neither.

- (a) $(p \vee q) \wedge (\neg p \vee q)$
 (b) $p \vee q \rightarrow \neg p \wedge q$
 (c) $p \rightarrow (\neg p \rightarrow q)$
 (d) $\neg(p \wedge q) \leftrightarrow \neg p \vee \neg q$

Logic Puzzles

1. The island of Knights and Knaves has only two kinds of inhabitants: Knights (who always tell the truth) and Knaves (who always lie). After being shipwrecked on the island, you decide to try to figure out who the Knights are so that you can ask them for help getting home. For each of the following groups you encounter, determine who the Knights and Knaves are:
- (a) Alice says "Bob and I are both Knights," and Bob says "Alice is a Knave"
 (b) Charlie says "I am a Knight or Dan is a Knave," Dan says "Charlie is a Knight and Edith is a Knave," Edith says "Dan and I are not the same"
 (c) Frank says "If you asked, George would tell you he's a Knave" then George says "No I wouldn't!"
2. While walking through the woods on the Island of Knights and Knaves you come to a fork in the trail. One path leads to a deep dark pit of doom and the other leads to a boat that can take you home. There are no signs indicating which direction to take, but there is one inhabitant of the island there waiting for you. Given that you don't know if he's a Knight or a Knave, what **one** question would you ask him to determine the correct path?
3. For their final exam, an evil math 55 professor has put a small mark on the forehead of 55 of his 555 students and will fail the entire class unless they can determine which of them has the mark. The students are not allowed to talk or communicate with each other in any way nor do they know how many of them were marked, but they can see each others foreheads. Once per minute, the Professor asks for all the marked students to come to the front of the room. If any unmarked students come, or if more than 1 hour passes, he will fail the entire class. At what minute do the 55 marked students turn themselves in?

Propositional Equivalence

- Show that $p \rightarrow q$ and $\neg q \rightarrow \neg p$ are logically equivalent by
 - Using a truth table
 - Using logical equivalence rules
- Without using truth tables, show that $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$ are logically equivalent.
- The operator NOR (abbreviated $p \downarrow q$) is defined to be true iff both p and q are false. For each of the following compound propositions, find an equivalent compound proposition that uses **only** the NOR operator:
 - $\neg p$
 - $p \wedge q$
 - $p \vee q$
- (a) Find a compound proposition that has the following truth table:

p	q	r	??
T	T	T	T
T	T	F	F
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	F
F	F	F	F

- Explain how you could generalize your procedure to any number of variables and any truth table. This is what we mean when we say \neg, \wedge, \vee are a *complete* set of connectives.