Name: Key

Math 55 Quiz 9
July 31, 2009
GSI: Rob Bayer

You have until 4:00 to complete this quiz. You must show your work.

1. (3 pts) A certain disease is present in 10% of the population. The test for this disease has the following properties: If you have the disease, the test will be positive 90% of the time. If you don't have the disease, the test will be negative (ie, correct) 80% of the time. If you test positive for the disease, what is the probability that you actually have it?

$$P(\text{have}|\text{pos}) = \frac{P(\text{pos}|\text{have})\cdot P(\text{have})}{P(\text{pos}|\text{have})\cdot P(\text{have}) + P(\text{pos}|\neg\text{have})\cdot P(\neg\text{have})}$$

$$= \frac{.9\cdot .1}{.9\cdot .1 + .2\cdot .9} = \cancel{\frac{9}{9+16}} = \cancel{\frac{9}{25}} = \cancel{36\%}$$

$$= \frac{1}{1+2} = \frac{1}{3}$$

2. (3 pts) Suppose you toss n balls into m bins uniformly at random. What is the expected number of bins that receive at least one ball?
Hint: create a r.v. for each bin whose value is based on whether it receives a ball or not.

Let $X_i = \begin{cases} 1 & \text{if } i \text{ gets a ball} \\ 0 & \text{o/w} \end{cases}$

Then $E[X_i] = 1\cdot P(i \text{ gets a ball}) = 1 - P(i \text{ doesn't})$
$= 1 - \left(\frac{m-1}{m}\right)^n$

Let $X = \#$ of bins w/ ≥ 1 ball. Then

$X = \sum_{i=1}^{m} X_i$, so $E[X] = \sum_{i=1}^{m} E[X_i] = m\left(1 - \left(\frac{m-1}{m}\right)^n\right)$

(over)

3. (4 pts) Consider the relation on $\mathbb{R}$ given by xRy iff $xy \geq 0$. Determine whether this relation has each of the following properties. You must justify your answers.

(a) Reflexive

Yes! $x \cdot x = x^2 \geq 0 \quad \forall x$ so xRx.

(b) Symmetric

Yes! If $xy \geq 0$, Then $yx \geq 0$ so

$xRy \Rightarrow yRx$

(c) Transitive

No! $3R0$ and $0R(-3)$, but

$3 \cdot (-3) = -9 < 0$, so

$3 \not R -3$

(d) Antisymmetric

No! $1R2$ and $2R1$ but ~~$1R1$~~

$1 \neq 2$