Name: _____

Math 55 Quiz 8 SOLUTIONS

July 31, 2009

GSI: Rob Bayer

You have until 4:00 to complete this quiz. You must show your work.

1. (3 pts) Suppose you generate a bit string of length 10 uniformly at random. What is the probability that it either starts with a 0 or ends with a 110?

   There are $2^9 + 2^7 - 2^6$ such strings, so the probability is

   $$\frac{2^9 + 2^7 - 2^6}{2^10} = \frac{9}{16}$$

2. (3 pts) Consider the experiment in which we roll two fair dice. Determine whether the events "the sum of the dice is even" and "the second is a 4" are independent.

   Let E_1 be the event "the sum is even" and E_2 be the event "the second is a 4"

   For each of the 6 possible numbers for the first die, there are 3 that will make the sum even, so $P(E_1) = \frac{18}{36} = \frac{1}{2}$

   Note $P(E_2) = \frac{1}{6}$

   $E_1 \cap E_2 = \{24, 44, 64\}$ so $P(E_1 \cap E_2) = \frac{3}{36} = \frac{1}{12}$

   So $P(E_1 \cap E_2) = P(E_1)P(E_2) = \frac{1}{12}$ and thus they are independent.

3. (4 pts) Suppose you flip a bias-p coin four times and it comes up heads exactly twice. What's the probability that the second flip was a tails? Recall that a bias-p coin is one that comes up heads with probability p.

   Let E be the event "It comes up head exactly twice" and F be the event "the second is a Tails." So we want $P(F|E) = \frac{P(E \cap F)}{P(E)}$

   By Bernoulli, $P(E) = \binom{4}{2} p^2(1-p)^2$.

   $E \cap F = \{HTHT, HTTH, TTHH\}$. Each outcome here has probability $p^2(1-p)^2$ so $P(E \cap F) = 3p^2(1-p^2)$ and we have

   $$P(F|E) = \frac{3p^2(1-p)^2}{6p^2(1-p)^2} = \frac{1}{2}$$