Name: ______________________________

Math 55 Quiz 6 SOLUTIONS

July 17, 2009
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You have until 4:00 to complete this quiz. You must show your work.

1. (3 pts) Find a recurrence relation for the number of ways to make n cents using only pennies and nickels:

   Any such sequence of change either is $n-1$ cents followed by a penny, or $n-5$ cents followed by a nickel. Thus, the recurrence relation is:

$$a_n = a_{n-1} + a_{n-5};\ a_0 = a_1 = a_2 = a_3 = a_4 = 1$$

2. (3 pts) Find the solution to the recurrence relation $a_n = 5a_{n-1} + 14a_{n-2};\ a_0 = 0, a_1 = 25$

   We solve $r^2 - 5r - 14 = 0$ and get $r = 7, -2$. Thus, the general solution is $c_1 7^n + c_2(-2)^n$.

   The initial conditions tell us that $c_1 + c_2 = 0$ and that $7c_1 - 2c_2 = 25$. Solving this gives $c_1 = \frac{25}{9}, c_2 = -\frac{25}{9}$

   Thus, the solution is

$$a_n = \frac{25}{9}(7^n - (-2)^n)$$

3. (4 pts) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2} + 4n;\ a_0 = 4, a_1 = 4$

   We'll start by solving $r^2 - 6r + 9 = 0$ and get $r = 3, 3$. Thus, the homogeneous part has solution $c_1 3^n + c_2 n 3^n$.

   We now guess $p_n = An + B$ and plug in:

$$\begin{aligned} An + B - 6(A(n-1) + B) + 9(A(n-2) + B) &= 4n \\ (A - 6A + 9A)n + (B + 6A - 6B - 18A + 9B) &= 4n \\ 4An + (4B - 12A) &= 4n \end{aligned}$$

   So $A = 1$ which makes $B = 3$. Thus, the general solution is $a_n = c_1 3^n + c_2 n 3^n + n + 3$.

   Plugging in initial conditions will give $4 = c_1 + 3$ so $c_1 = 1$ and $4 = 3c_1 + 3c_2 + 4$, so $c_2 = -1$.

   Thus, the solution is

$$a_n = 3^n - n3^n + n + 3$$