Name: _____

Math 55 Quiz 4 SOLUTIONS
July 6, 2009
GSI: Rob Bayer

You have until 4:00 to complete this quiz. You must show your work.

1. (3 pts) Prove that if $a \equiv b$ (**mod** m) and $n|m$, then $a \equiv b$ (**mod** n)

   Since $a \equiv b$ (**mod** m), we know $m|a-b$ so $a-b=km$ for some $k \in \mathbb{Z}$. Since $n|m, m=nl$ for some $l \in \mathbb{Z}$. Combining these gives $a-b=knl=(kl)n$ so $n|a-b$. Thus, $a \equiv b$ (**mod** n)

2. (3 pts) Prove that there are no solutions in positive integers to $x^3 \equiv 2$ (**mod** 4)

   Since we're working mod 4, we need only check $x=0,1,2,3$:

$$x=0: \ 0^3=0 \not\equiv 2$$
$$x=1: \ 1^3=1 \not\equiv 2$$
$$x=2: \ 2^3=8 \not\equiv 2$$
$$x=3: \ 3^3=27 \not\equiv 2$$

3. (4 pts) Determine whether each of the following sets are countable or not. For those that are, **briefly** describe why.

   (a) The set of all even integers

   This is a subset of a countable set and thus is countable.

   (b) The set of all irrational numbers

   Uncountable. (To see this, note that $\mathbb{R} = (\mathbb{R}-\mathbb{Q}) \cup \mathbb{Q}$ so the irrationals must be uncountable since $\mathbb{R}$ is.)

   (c) The set of all English language sentences

   We can list all the English languate sentences by first listing all the ones of length 1 in alphabetical order, then all the ones of length 2, then all those of length 3, etc. Since every sentence has finite length, we will eventually list all sentences and thus our listing is a surjection from $\mathbb{N}$ to this set. Thus, this set is countable.

   (d) The set of all real numbers that have no 1's in their decimal representation.

   Uncountable. The same diagonalization argument we used for $\mathbb{R}$ works here