Name: ______________________________

Math 55 Quiz 3
July 1, 2009
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You have until 4:00 to complete this quiz. You must show your work.

1. (3 pts) Prove that the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = 3x - 4$ is a bijection.

   (injective) If $f(x) = f(y)$, then $3x - 4 = 3y - 4$ so adding 4 gives $3x = 3y$ and thus $x = y$. Therefore, $f(x) = f(x) \Rightarrow x = y$ and f is injective.

   (surjective) Let $y \in \mathbb{R}$. Then $\frac{y+4}{3} \in \mathbb{R}$ and $f(\frac{y+4}{3}) = 3\frac{y+4}{3} - 4 = y$, so $\forall y \exists x f(x) = y$ and f is surjective.

   $\therefore f$ is a bijection

2. (3 pts) Let $f : D \to E$ be a function and let $A, B \subseteq D$. Prove that $f(A \cap B) \subseteq f(A) \cap f(B)$

   (Proof 1) Let $y \in f(A \cap B)$. So $\exists x \in A \cap B$ such that $f(x) = y$. Since $x \in A \cap B$, $x \in A$, so $y = f(x) \in f(A)$. Similarly, $x \in B$ so $y \in f(B)$. Thus, $y \in f(A) \cap f(B)$.

   (Proof 2) $A \cap B \subseteq A$ so $f(A \cap B) \subseteq f(A)$. Similarly, $f(A \cap B) \subseteq f(B)$. Thus, $f(A \cap B) \subseteq f(A) \cap f(B)$

3. (4 pts) Show that it is not possible to tile a 6×6 chessboard with three corners removed using 1×3 rectangles

   We'll color the board as follows:

X	R	G	B	R	X
G	B	R	G	B	R
R	G	B	R	G	B
B	R	G	B	R	G
G	B	R	G	B	R
R	G	B	R	G	X

   Note that anywhere you put a 1×3 triominoe on this board, it will cover exactly one red, one green, and one blue square. Since there are 33 squares to cover, any covering must use 11 triominoes. However, a quick count shows that there are 12 red squares, and thus they cannot all be covered by just 11 triominoes. Thus, there is no possible tiling of this board.