Name: _____

Math 55 Quiz 2 SOLUTIONS

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You have until 4:00 to complete this quiz. You must show your work.

1. (3 pts) Re-write the statement $\neg\forall x\exists y(P(x) \wedge \neg Q(y))$ so that negations appear only directly in front of predicates.

$$\boxed{\exists x\forall y(\neg P(x) \vee Q(y))}$$

2. (3 pts) Prove that $m^2 = n^2$ if and only if $m = n$ or $m = -n$

   We'll do this by showing both directions individually:

   ($\Rightarrow$) Suppose $m^2 = n^2$. Then $m^2 - n^2 = 0$, so $(m-n)(m+n) = 0$. Thus, either $m = n$ or $m = -n$

   ($\Leftarrow$) Suppose $m = n$ or $m = -n$. Then if $m = n$, squaring both sides gives $m^2 = n^2$.

   If $m = -n$, then squaring both sides gives $m^2 = (-n)^2 = (-1)^2n^2 = n^2$.

   In either case, $m^2 = n^2$, so we conclude this direction is true.

$$\therefore m^2 = n^2 \Leftrightarrow m = n \text{ or } m = -n$$

3. (4 pts) Prove that if $x + y$ is irrational, then either x is irrational or y is irrational.

   We'll do this by contrapositive. That is, we'll show that

$$x \in \mathbb{Q} \text{ and } y \in \mathbb{Q} \Rightarrow x + y \in \mathbb{Q}$$

   Suppose $x, y \in \mathbb{Q}$ and let $a, b, c, d \in \mathbb{Z}$ such that $x = \frac{a}{b}, y = \frac{c}{d}$. Then

$$x + y = \frac{a}{b} + \frac{c}{d} = \frac{ad + cb}{bd} \in \mathbb{Q} \text{ (since } ad + cb, bd \in \mathbb{Z})$$

   Thus, we have the desired result.