

Name: Key

Math 55 Quiz 1

June 24, 2009

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You have until 4:00 to complete this quiz. You must show your work.

1. (3 pts) Construct a truth table for the compound proposition  $(p \rightarrow \neg q) \wedge (q \leftrightarrow \neg p \vee q)$

$p$	$q$	$p \rightarrow \neg q$	$\neg p \vee q$	$q \leftrightarrow (\neg p \vee q)$	$\downarrow$
T	T	F	T	T	F
T	F	T	F	T	T
F	T	T	T	T	T
F	F	T	T	F	F

2. (3 pts) Without using Truth Tables, show that  $(p \rightarrow r) \vee (q \rightarrow r) \equiv p \wedge q \rightarrow r$ .

$$\begin{aligned}
 (p \rightarrow r) \vee (q \rightarrow r) &\equiv (\neg p \vee r) \vee (\neg q \vee r) \\
 &\equiv \neg p \vee \neg q \vee r \\
 &\equiv \neg (p \wedge q) \vee r \\
 &\equiv p \wedge q \rightarrow r
 \end{aligned}$$

(over)

3. (4 pts) Let  $S(x)$  be "x is a student,"  $P(x)$  be "x is a professor,"  $T(x, y)$  be "x is teaching y,"  $E(x, y)$  be "x is enrolled in y," and  $L(x, y)$  be "x likes the class y." If the domains are all people and all classes at UC Berkeley (as appropriate), translate each of the following between English and Logic:

- (a) "Every Student is enrolled in at least one class"

~~$\forall x (S(x) \rightarrow \exists y E(x, y))$~~

$$\forall x (S(x) \rightarrow \exists y E(x, y))$$

- (b) "Professors only teach classes they enjoy"

$$\forall x \forall y (P(x) \wedge T(x, y) \rightarrow L(x, y))$$

- (c)  $\exists y \forall x [(P(x) \rightarrow L(x, y)) \wedge (S(x) \rightarrow \neg L(x, y))]$

There is a class that all professors like but all students dislike.

- (d)  $\exists x [P(x) \wedge \forall y \forall z (S(z) \wedge T(x, y) \rightarrow L(z, y))]$

~~There~~ There is a ~~professor~~ professor whose classes are liked by all students.