§5.2#36 Prove that at a party with $n \geq 2$ people, at least 2 know the same number.
Suppose not. Then since each person can only know 0, 1, 2, ... n-1 other people, there must be someone who knows 0 people and someone who knows $n-1$ people (otherwise, n people would have $n-1$ different numbers of known people and by the PHP two would be the same). However, this is not possible since you cannot have somebody who knows nobody and somebody who knows everyone at the same party.

§5.4#22 Prove that $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$

(a) Using a combinatorial argument.

Suppose we want to make an r-memeber comittee from a group of n people, and we want to designate k of these r people as Executive Board members. Then the LHS counts the number of possible committees since $\binom{n}{r}$ counts how many comittees you can make and the $\binom{r}{k}$ counts the number of ways you can pick the Executive.

The RHS also counts this because the $\binom{n}{k}$ counts the number of ways to pick the Exec Board and the $\binom{n-k}{r-k}$ counts how many ways to fill-in the rest of the committee with non-Exec Board members.

(b) By using the formula for $\binom{n}{k}$

$$\begin{aligned}
LHS &= \binom{n}{r}\binom{r}{k} \\
&= \frac{n!}{r!(n-r)!}\frac{r!}{k!(r-k)!} \\
&= \frac{n!}{(n-r)!(r-k)!k!} \\
RHS &= \binom{n}{k}\binom{n-k}{r-k} \\
&= \frac{n!}{k!(n-k)!}\frac{(n-k)!}{(r-k)!(n-k-(r-k))!} \\
&= \frac{n!}{k!(n-r)!(r-k)!}
\end{aligned}$$

So the two are equal.