§7.1#42

(a) Find a recurrence relation for the number of tiling of a $2 \times n$ board using dominoes.

Consider the upper right corner. If it's covered by a vertical tile, then this is a tiling of a $2 \times (n-1)$ board followed by a vertical dominoe. Otherwise, the two tiles below it must also be covered by a horizontal dominoe and thus this is a $2 \times (n-2)$ tiling followed by 2 horizontal dominoes. Thus, the recurrence relation is:

$$a_n = a_{n-1} + a_{n-2};\ a_0 = 1, a_1 = 1$$

(b) See above

(c) This is the same as the 18th fibonacci number, which is 2584

§7.2#28 Find all solutions to $a_n = 2a_{n-1} + 2n^2$

We'll start by solving the homogeneous part $a_n = 2a_{n-1}$ which has characteristic equation $r - 2 = 0$ and thus the general solution is $a_n = C2^n$. For the $2n^2$ part, we guess $p_n = An^2 + Bn + C$:

$$\begin{aligned} An^2 + Bn + C &= 2(A(n-1)^2 + B(n-1) + C) + 2n^2 \\ An^2 + Bn + C - 2A(n^2 - 2n + 1) - 2B(n-1) - 2C &= 2n^2 \\ (A - 2A)n^2 + (B + 4A - 2B)n + (C - 2A + 2B - 2C) &= 2n^2 \end{aligned}$$

So $-A = 2$ and $A = -2$. Then $-B - 8 = 0$, so $B = -8$. Then $-C + 4 - 16 = 0$ so $C = -12$.
Thus, the general solution is

$$a_n = C2^n - 2n^2 - 8n - 12$$

If we also know $a_1 = 4$, then we have $4 = 2C - 2 - 8 - 12$ and thus $C = 13$. Final solution:

$$13 \cdot 2^n - 2n^2 - 8n - 12$$