§2.4#34

(a) The integers not divisible by 3 are countable. We use the same correspondence as for all the integers, but we skip those that are divisible by 3.

(b) The integers divisible by 5 but not 7 are countable. We use the same correspondence as above, but skip all those integers not divisible by 5 and all those that are divisible by 7.

(c) The real numbers with only 1's in its decimal rep is countable. Given any real number of the form $\underbrace{11\ldots1}_{n}.\underbrace{11\ldots1}_{m}$, we can map it to the pair (n, m). Anything ending in infinitely many 1's can be handled by mapping to another copy of $\mathbb{N}$ based on integer part. Thus, we have a bijection between this set and $(\mathbb{N} \times \mathbb{N}) \cup \mathbb{N}$ which is countable.

(d) the set of reals with only 1's and 9's in their decimal representations is uncountable. We can see this by either a) using the same diagonalization argument as in class or by b) replacing 9's with 0's and interpreting the numbers in binary, thus creating a bijection between this set and the set of all reals.

§3.4#22 Show that if $a \equiv b \pmod{m}$, then $ac \equiv bc \pmod{mc}$
If $a \equiv b \pmod{m}$, then $m|a - b$, so $a - b = km$ for some $k \in \mathbb{Z}$. Note then that $ac - bc = (a - b)c = kmc$ so $mc|ac - bc$ and thus $ac \equiv bc \pmod{mc}$