Math 55 Homework 2 SOLUTIONS

§1.7#14 Show that for all $a, b, c \in \mathbb{R}$ with $a \neq 0$, there is a unique solution to $ax + b = c$.
(existence): Since $a \neq 0$, $\frac{c-b}{a} \in \mathbb{R}$ and $a\frac{c-b}{a} + b = c - b + b = c$, so there is at least one solution.
(uniqueness): Let x and y be any two solutions. Then $ax + b = c$ and $ay + b = c$ so in particular $ax + b = ay + b$. By subtracting b we get $ax = ay$. Since $a \neq 0$, we can divide by a to get $x = y$. Thus, there is only one solution.

§2.3#30 If f and $f \circ g$ are injective, does it follow that g is injective?
Yes! We'll show the contrapositive. That is, we'll show that if g is not injective, then $f \circ g$ is not injective.
If g is not injective, then $\exists x \neq y$ such that $g(x) = g(y)$. Then applying f to both sides, we get $f(g(x)) = f(g(y))$, which is the same as $(f \circ g)(x) = (f \circ g)(y)$ and thus $f \circ g$ is not injective.