Math 1B PDP Worksheet

Variation of Parameters

1. For each of the following problems, decide whether you'd use undetermined coefficients or variation of parameters. For those where you'd use variation of parameters, **solve the problem**.
   (a) $y'' - y = \frac{1}{x}$
   (b) $y'' + y = e^x \sin x$
   (c) $y'' + 4y' + 5y = e^{-2x} \tan x$
   (d) $y'' + 2y' + y = 3x^2 + \cos 2x$
   (e) $y'' + 2y' + y = \frac{e^x}{1+x^2}$
   (f) $y'' + 4y' + 3y = \frac{1}{1+e^{2x}}$
2. Use a combination of variation of parameters and undetermined coefficients to find the general solution to $y'' + y = e^x + \sec^2 x$
3. Show that when using variation of parameters, the answer is always

$$u_1' = -\frac{G(x)y_2}{a(y_1y_2' - y_1'y_2)} \qquad u_2' = \frac{G(x)y_1}{a(y_1y_2' - y_1'y_2)}$$

   Note: you may NOT use this fact on quizzes or the final. You must write down the system of equations and solve it each time.

Extra Problems

1. Find a Macluarin series for $f(x) = \sin x \cos x$
2. When taking derivative and integrals of functions involving complex numbers, we just do the same thing as for real-valued functions, treating complex constants the same as real valued constants.
   (a) Find $\frac{d}{dx}e^{(a+bi)x}$
   (b) Remembering that integrals just signify anti-derivatives, find $\int e^{(1+i)x}dx$. By properly calculating $\frac{1}{1+i}$, (hint: multiply top and bottom by the conjugate) re-write your answer in the form $f(x) + ig(x)$
   (c) By equating real and imaginary parts and using Euler's formula, use your answer to (b) to find $\int e^x \cos x dx$ and $\int e^x \sin x dx$
3. Find the sum: $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{2 \cdot 4 \cdot 6 \cdot 8 \cdots (2n)}$