**Finding Taylor Series**

1. Find a Taylor Series for each of the following functions centered at the given $a$. Be sure to also find the radius and interval of convergence.

   (a) $\ln(2x+3)$; $a=2$

   (b) $\dfrac{1}{3+x}$; $a=-5$

   (c) $\dfrac{x}{(3x-1)^2}$; $a=2$

   (d) $e^x$; $a=-1$

   (e) $\displaystyle\int_0^x e^{t^2}dt$; $a=0$

2. Consider the function $f(x)=\begin{cases} e^{-1/x^2} & x\neq 0 \\ 0 & x=0 \end{cases}$

   (a) Show that $f'(0)$ and $f''(0)$ are both 0.

   (b) Convince yourself that $f^{(n)}(0)=0$ for all $n$. What does this mean the Maclaurin series for $f$ must be?

   (c) What is its radius and interval of convergence?

   (d) For what values of $x$ does it converge to $f(x)$?

**Finding Sums**

1. Find a closed form (ie, no $\sum$ or "$\cdots$") for each of the following. For those that include an $x$, your answer should be a function. For those without an $x$, you answer should be a number.

   (a) $\displaystyle\sum_{n=1}^{\infty} nx^{n-1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{3^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} n^2x^n$

   (d) $\displaystyle\sum_{n=1}^{\infty} (-1)^n\frac{n^2}{2^{2n+3}}$

   (e) $\displaystyle\sum_{n=0}^{\infty} (-1)^n\frac{x^n}{n+1}$

   (f) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+1}{3^n}$

   (g) $\displaystyle\sum_{n=1}^{\infty} n(n+1)x^{n+2}$

   (h) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{2^{n+2}(n+2)(n+1)}$