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Math 1B PDP Worksheet

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Power Series

1. Find the radius and interval of convergence for each of the following power series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n2^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x+1)^n}{\sqrt[3]{n+2}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(3x-2)^n}{(n+2)(n-3)}$

   (d) $\displaystyle\sum_{n=1}^{\infty} n!(x-1)^n$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^n}$

   (f) $\displaystyle\sum_{n=1}^{\infty} x^n \ln\left(1+\frac{1}{n}\right)$

2. Prove that if $\lim_{n\to\infty} \sqrt[n]{|c_n|}$ converges to a value c, then the radius of convergence of $\sum c_n x^n$ is $\frac{1}{c}$

3. True/False. Justify your answer with a proof or a counterexample.

   (a) If $\sum c_n 2^n$ converges, then $\sum c_n(-2)^n$ converges

   (b) If $\sum c_n(-4)^n$ converges, then $\sum c_n 3^n$ converges

   (c) If $\sum c_n(-4)^n$ diverges, then $\sum c_n 3^n$ diverges

   (d) If $\sum c_n$ converges and $|x| < 1$, then $\sum c_n x^n$ converges

   (e) If $\sum c_n x^n$ has positive radius of convergence, then $\lim c_n = 0$

   (f) Given the interval of convergence of a power series, you can determine the radius of convergence.

   (g) Given the radius of convergence of a power series, you can determine the interval of convergence.

4. We said that the ratio test gives no information when $L = 1$. Prove this by giving an example of a series with $\lim\left|\frac{a_{n+1}}{a_n}\right| = 1$ and

   (a) $\sum a_n$ converges absolutely

   (b) $\sum a_n$ converges conditionally

   (c) $\sum a_n$ diverges

5. Prove that if $\sum a_n$ converges and $|x| < 1$, then $\sum a_n x^n$ converges absolutely.