

**Root/Ratio Tests**

1. Determine whether each of the following series is absolutely convergent, conditionally convergent, or divergent:

(a)  $\sum_{n=1}^{\infty} \frac{2^n}{n!}$

(f)  $\sum_{n=1}^{\infty} \frac{2^n n!}{n^n}$

(b)  $\sum_{n=1}^{\infty} \frac{n3^n}{5^{n-1}}$

(g)  $\sum_{n=1}^{\infty} \frac{2 \cdot 4 \cdot 6 \cdots 2n}{(2n)!}$

(c)  $\sum_{n=1}^{\infty} \left(\frac{\ln n}{n}\right)^n$

(h)  $\sum_{n=1}^{\infty} \left(\frac{n+1}{n}\right)^{3n} \frac{1}{3^n}$

(d)  $\sum_{n=1}^{\infty} \frac{(n^4 + 1)2^{2n}}{5^{n+3}}$

(i)  $\sum_{n=1}^{\infty} \left(\frac{n}{4n-1}\right)^{2n+1}$

(e)  $\sum_{n=1}^{\infty} \left(\frac{n-1}{n+1}\right)^{n^2}$

2. We said that the ratio test gives no information when  $L = 1$ . Prove this by giving an example of a series with  $\lim \left| \frac{a_{n+1}}{a_n} \right| = 1$  and

(a)  $\sum a_n$  converges absolutely

(b)  $\sum a_n$  converges conditionally

(c)  $\sum a_n$  diverges

3. Let  $p(n)$  be a polynomial of degree  $l$  and  $q(n)$  be a polynomial of degree  $k$ . Show that regardless of  $l$  and  $k$ , the ratio test always gives  $L = 1$  when applied to  $\sum \frac{p(n)}{q(n)}$ . What conditions on  $l$  and  $k$  make this converge? Diverge? Hint: if  $l$  and  $k$  are too abstract, try a few examples with actual polynomials and see what happens.

4. True/False. Prove or provide counterexamples.

(a) If  $L = 1$  in the ratio test, then  $\sum a_n$  is conditionally convergent.

(b) If  $\sum a_n$  is conditionally convergent, then  $\lim \left| \frac{a_{n+1}}{a_n} \right|$  either DNE or is 1.

(c) If  $\sum a_n$  and  $\sum b_n$  are divergent series with only non-negative terms, then  $\sum (a_n + b_n)$  diverges.

(d) If  $\sum a_n$  converges and  $a_n > 0$ , then  $\sum (-1)^n a_n$  converges

5. (A preview of power series)

(a) For which values of  $x$  does  $\sum_{n=0}^{\infty} \frac{x^n}{n}$  converge? Hint: ratio test. What do you do with the  $x$ 's that give you 1?

(b) For which values of  $x$  does  $\sum_{n=0}^{\infty} (-1)^n \frac{n^2(x-1)^n}{3^n}$  converge? Hint: see above

6. For what values of  $r$  and  $p$  will  $\sum r^n n^p$  converge? What about  $\sum r^n (n!)^p$ ?

7. Prove that if  $\sum a_n$  converges and  $|x| < 1$ , then  $\sum a_n x^n$  converges absolutely.