Math 1B PDP Worksheet

More Sequences

1. Determine whether each of the following sequences are monotone, bounded, both, or neither.

   (a) $\dfrac{1}{3n-4}$

   (b) $\sin(\dfrac{1}{n})$

   (c) $\dfrac{3n-2}{n+1}$

2. Give an example of each of the following:

   (a) A bounded sequence that diverges

   (b) A monotone sequence that diverges

   (c) A convergent sequence that isn't monotone

3. In addition to defining sequences by a formula of the form $a_n = f(n)$, we can also define a sequence by giving the value of a_1 and a rule for getting the next term in the sequence from the previous one. For example, we could say $a_n = 3 + a_{n-1}; a_1 = 2$. This then defines the sequence $a_1 = 2, a_2 = 3 + a_1 = 3 + 2 = 5, a_3 = 3 + a_2 = 8, a_4 = 3 + a_3 = 11$, etc. Such a definition is called recursive. A definition that doesn't refer to previous terms or have any "..."s in it is called a closed form. For example, a closed form for the sequence above would be $a_n = 2 + 3(n-1)$

   For each of the following recursively defined sequences, find a closed form for a_n. Use your answer to determine if each is convergent or divergent.

   (a) $a_n = na_{n-1};\ a_1 = 1$

   (b) $a_n = a_{n-1} + n;\ a_1 = 1$

   (c) $a_n = ra_{n-1};\ a_1 = a$ where a, r are contants and $|r| < 1$

   (d) $a_n = \ln n + a_{n-1};\ a_1 = 0$

Series I

1. Re-write each of the following in $\sum$ notation. Start your sum wherever is convenient.

   (a) $1 + 2 + 3 + \cdots$

   (b) $1 + \frac{1}{4} + \frac{1}{9} + \cdots$

   (c) $1 - 1 + 1 - 1 + \cdots$

   (d) $\frac{3}{2} - \frac{3}{4} + \frac{3}{8} - \frac{3}{16} + \cdots$

2. Re-index each of the following series to start at $n = 0$

   (a) $\displaystyle\sum_{n=1}^{\infty} ar^{n-1}$

   (b) $\displaystyle\sum_{n=2}^{\infty} \ln\left(\frac{n+1}{n+2}\right)$

   (c) $\displaystyle\sum_{n=-1}^{\infty} \sin^n\left(\frac{n\pi}{2}\right)$

3. Determine whether each of the following series are convergent or divergent. For those that are convergent, find the sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n+2}}{2^{2n}}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \sqrt[n]{2}$

   (c) $\displaystyle\sum_{n=0}^{\infty} (-1)^n \frac{2+3^n}{4^n}$

   (d) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2-3}{n^2+3n+1}$

4. By using partial sums (ie, the definition of a series), determine whether each of the following converge or diverge. For those that converge, find the sum:

   (a) $\displaystyle\sum_{n=1}^{\infty} \sin(n+1) - \sin(n)$

   (b) $\displaystyle\sum_{n=2}^{\infty} \ln\frac{n^2+2n+1}{n^2}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{-2}{n(n+2)}$

5. If the nth partial sum of a series $\displaystyle\sum_{n=1}^{\infty} a_n$ is $s_n = \dfrac{n-1}{n+1}$, find a_n and $\displaystyle\sum_{n=1}^{\infty} a_n$

6. For which values of x do each of the following converge?

(a) $\sum_{n=0}^{\infty} x^n$

(b) $\sum_{n=0}^{\infty} x^n 2^n$

(c) $\sum_{n=1}^{\infty} \frac{x}{n}$

(d) $\sum_{n=0}^{\infty} \frac{(x+1)^n}{3^{2n+1}}$

7. True/false. For those that are true, provide a brief explanation/intuition of why. For those that are false, find a counterexample:

   (a) If a_n is positive for all n, and each partial sum is less than 10^4, then $\sum_{n=0}^{\infty} a_n$ converges

   (b) If $a_n < b_n$ for all n and both sequences converge, then $\lim a_n < \lim b_n$

   (c) If s_n is the sequence of partial sums for the sequence a_n and $\lim_{n\to\infty} a_n = 0$, then $\lim_{n\to\infty} s_n$ exists.