Math 1B PDP Worksheet

Intro to Differential Equations

1. For each differential equation, determine if the given function is a solution:
   (a) $y' = e^x + y$; $y = xe^x$
   (b) $\frac{dP}{dx} = 1 + P^2$; $P = \tan x$
   (c) $(y')^2 = 4 + y^2$; $y = e^x - e^{-x}$
   (d) $f' = \frac{1}{e^f}$; $f = \ln(x + C)$

2. (a) Find the general solution for each of the following differential equations:
      i. $\frac{dy}{dx} = e^x$
      ii. $y'' = 20x^3 + 2$
   (b) Solve the initial value problem $y' = e^x$, $y(0) = 3$
   (c) How many initial conditions would you need in order to solve an initial value problem for part (ii)?

3. Show that $y = C\sec x + C\tan x$ is a 1 parameter family of solutions to $y' = y\sec x$. If $y(\pi/4) = 5$, what is C?

4. Find an equation for the curve that passes through the point $(0, 1)$ and whose slope at the point (x, y) is y.

Separable Equations

1. Solve each of the following separable equations/initial value problems:
   (a) $y' = xe^y$
   (b) $\dfrac{dy}{dx} = e^{x+y}(e^x + 1)^{-1}$
   (c) $y' = \dfrac{y\cos x}{1 + y^2}$; $y(0) = 1$
   (d) $y' = \dfrac{e^y \sin^2\theta}{y\sec\theta}$
   (e) $y' = e^x + e^x y^2$
   (f) $y' = 2 + 2y + t + ty$; $y(0) = -1$

2. In Math 1A, we talked about Newton's Law of Cooling, which basically says that $T' = k(T - T_s)$ where $T(t)$ is the tempurature of some object at time t and T_s is the (constant) surrounding temperature. Solve this differential equation and show you get the formula you were told to memorize in 1A.
   Hint: to get the familiar form, solve with the initial value $T(0) = T_0$

3. The differential equation $\dfrac{dy}{dx} = ky^{1+a}, a > 0$ is sometimes called the Doomsday Equation. Here we'll try to figure out why:
   (a) Solve this equation in terms of k, a, and an arbitrary constant C. How is this fundamentally different than the solution to $y' = ky$?
   (b) Now suppose the growth of a population of rabbits follows the differential equation $y' = 2y^{1.01}$ and suppose there are initially only two rabbits. When should we all start running for the hills?

4. One model for an falling object that encounters wind resistance is $a = g - bv^2$. Solve this equation for the $x(t)$, the height of the object at time t. You may use the initial conditions $v(0) = x(0) = 0$.
   Hint: By definition, $a = v'$ and $v = x'$.