

Arc Length

1. What is the length of the curve $y = \frac{x^2+1}{2}$ between $x = 0$ and $x = 2$?
2. Prove that the circumference of a circle is $2\pi r$
3. (a) Sketch the graph of the curve $f(x) = x \sin(1/x)$ for $0 < x \leq 1$
(b) Is the arc length of this curve finite or infinite? Prove your answer.
4. Let $a < b$. Show that the shortest differentiable path between the points $(a, 0)$ and $(b, 0)$ is a straight line.

Surface area

1. Find the surface area of the solid obtained by rotating $y = \sqrt{x}$ about the x-axis for $\frac{3}{4} < x < \frac{15}{4}$
2. Find the surface area of the solid obtained by rotating $y = \sqrt[3]{x}$ about the y-axis for $1 \leq y \leq 2$ (Note: this interval is given in terms of y , so be careful)
3. Prove that the surface area of a sphere is $4\pi r^2$
4. Consider the function $f(x) = 1/x$ for $1 \leq x < \infty$
 - (a) Show that the area under this curve is infinite
 - (b) Show that the length of this curve is infinite
 - (c) If you rotate this curve around the x-axis, you get a solid known as Gabriel's Horn or Torcelli's Trumpet. Sketch a picture of the trumpet and show that its surface area is infinite.
 - (d) Show that despite parts (a) - (c), the **volume** of Gabriel's Horn is actually finite.
 - (e) What does this mean in terms of being able to fill it with paint vs being able to paint the inside?
5. A sphere can be thought of as a half-circle rotated around the x-axis. In the same way, rotating a half-ellipse gives something called an ellipsoid. Set up, but do not attempt to evaluate, the integral for the surface area of the ellipsoid you get by rotating the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ around the x-axis.

Extra Problems If you finish early, take a stab at these

1. Determine whether $\int_{\pi}^{\infty} \sin^3 x \, dx$ converges or diverges
2. For what values of r does $\int_1^{\infty} r^x \, dx$ converge?
3. Consider the arclength function $s(x) = \int_a^x \sqrt{1 + (f'(t))^2} \, dt$.
 - (a) What is the minimum value for $s'(x)$?
 - (b) Does this make sense geometrically?