Math 1B PDP Worksheet

Comparison Tests

1. Determine whether each of the following integrals converges or diverges.

(a) $\displaystyle\int_0^1 \csc^2 x\sqrt{\ln(1+x)}\,\mathrm{d}x$

(b) $\displaystyle\int_{10}^\infty \frac{\sqrt{x^3-3}+\sqrt[3]{x^4+3x^2+2}}{\sqrt[3]{x^8+3x-2}}\,\mathrm{d}x$

(c) $\displaystyle\int_0^\infty \frac{\mathrm{d}x}{(\tan^{-1}x)(5^x-1)}$ (be careful...)

(d) $\displaystyle\int_3^\infty \tan^{-1}(\ln(x^2+1)-2\ln(x))\,\mathrm{d}x$

(e) $\displaystyle\int_1^\infty \frac{\sin^2 x}{x^3}\,\mathrm{d}x$

(f) $\displaystyle\int_0^1 \frac{\mathrm{d}x}{6^x-2^x-3^x+1}$

(g) $\displaystyle\int_0^\infty \frac{\mathrm{d}x}{x+e^x}$

2. Sometimes the function you're given is hard to apply the comparison or limit comparison tests to because it is not always positive. One way around this is to use the following theorem:

$$\text{If } f(x) \text{ is continuous on } (a,b) \text{ and } \int_a^b |f(x)|\,\mathrm{d}x \text{ converges, then } \int_a^b f(x)\,\mathrm{d}x \text{ does too.}$$

Here we'll show how this theorem can be used to show that $\int_0^\infty \frac{\sin x}{x}\,\mathrm{d}x$ converges.

(a) Argue that $\displaystyle\int_0^1 \frac{\sin x}{x}\,\mathrm{d}x$ is not even improper, so we need only test $\displaystyle\int_1^\infty \frac{\sin x}{x}$

(b) Use integration by parts and the theorem above to show that $\displaystyle\int_1^\infty \frac{\sin x}{x}$ converges

3. Consider the integral $\displaystyle\int_1^\infty \frac{\sin^2 x}{x^2}\,\mathrm{d}x$ Show that this integral converges

(a) By using the regular comparison test

(b) By using the limit comparison test

(c) Which was easier?

Integration Practice

1. Find each of the following integrals. Be sure to work as a group so **everyone** knows how to do all these problems.

(a) $\displaystyle\int e^{x+e^x}dx$

(b) $\displaystyle\int \frac{\sec^2(\sin\theta)}{\sec\theta}d\theta$

(c) $\displaystyle\int \frac{1}{\sqrt{x+1}+\sqrt{x}}dx$

(d) $\displaystyle\int \frac{\ln(x+1)}{x^2}dx$

(e) $\displaystyle\int \frac{t^3+1}{t^3-t^2}dt$

(f) $\displaystyle\int \cos^3 2x\sin 2x dx$

(g) $\displaystyle\int (\sin^{-1}x)^2dx$

(h) $\displaystyle\int \frac{dt}{\sqrt{e^t}}$

(i) $\displaystyle\int \frac{1}{x\sqrt{x^2+4}}dx$

(j) $\displaystyle\int \frac{1}{x\sqrt{x+4}}dx$

(k) $\displaystyle\int \frac{x}{\sqrt{x^2+4}}dx$

(l) $\displaystyle\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}$

(m) $\displaystyle\int \ln(\sec\theta)\sec^2\theta d\theta$