Instructions

1. Introduce yourselves!
2. Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
3. Do the problems below, having a different person be the scribe for each one.

Partial Fractions

1. Each of the following represent a potential term in a partial fraction decomposition. Find each integral:

   (a) $\int \frac{dx}{(2x-3)^3}$ (b) $\int \frac{x-1}{(x^2+2x+3)^2}dx$

2. Give the form of the partial fraction decomposition for each of the following. You need not solve for A, B, C, etc.

   (a) $\frac{x^3+2x-1}{x^4-3x^3-4x^2}$ (b) $\frac{3x^2-7x}{(x-2)^2(x^2+x+1)^2x^3}$ (c) $\frac{x^3-1}{x^2-x}$

3. Consider the integral $\int \frac{dx}{x^2-4}$. Find this integral
   (a) Using partial fractions
   (b) Using trig substitution
   (c) Use log laws to show that your answers are the same

4. Find each of the following:

   (a) $\int \frac{6x^3+7x^2-2x-5}{x^4-x^2}dx$ (b) $\int \frac{x^4+1}{x(x^2+1)^2}dx$

5. Find $\int \frac{3e^{2t}}{e^{2t}-e^t-6}dt$

6. Find $\int \frac{dx}{x^4+1}$ Hint: add and subtract $2x^2$ in the denominator and factor

Extra Problems If you finish early, take a stab at these.

1. (The Weirstrass Substitution) It turns out that **any** rational function of sin and cos (and hence, of $\sec, \tan, \csc, \cot$, etc) can be turned into an ordinary rational function via the substitution $t = \tan(\frac{x}{2})$. Let's explore why:
   (a) Show that $\cos\left(\frac{x}{2}\right) = \frac{1}{\sqrt{t^2+1}}$ and that $\sin\left(\frac{x}{2}\right) = \frac{t}{\sqrt{t^2+1}}$. Hint: right triangles
   (b) Use trig identities to show that $\cos x = \frac{1-t^2}{1+t^2}$, and that $\sin x = \frac{2t}{1+t^2}$
   (c) Show that $dx = \frac{2}{1+t^2}dt$
   (d) Use parts (b) and (c) to find $\int \frac{dx}{3\sin x - 4\cos x}$ and $\int \sec^3 x dx$

2. Find each of the following. While you could use the technique of the previous problem for (a) and (b), you should try to find a more straightforward way.

   (a) $\int \frac{\cos x - 1}{\cos x + 1}dx$ (b) $\int \frac{\sin x + \cos x}{\sin 2x}dx$ (c) $\int \frac{dx}{\sqrt{x}+\sqrt[3]{x}}$

3. Let S be a sphere of radius 1. Now take a plane a distance $d < 1$ away from the center and slice the sphere into two pieces. What is the volume of each piece? Hint: the formula for the volume of a revolved solid is $\int_a^b \pi f(x)^2$