Instructions

1. Introduce yourselves!
2. Find some blackboard space, a piece of chalk, and decide who will be your first scribe.
3. Do the problems below, having a different person be the scribe for each one.

Trig Substitutions

1. Consider the integral $\int \frac{x}{\sqrt{9-x^2}}dx$
   (a) Find this integral using an appropriate trig substitution
   (b) Find this integral using an ordinary u-substitution
   (c) Are your answers the same?
2. Find each of the following:
   (a) $\int \frac{dx}{x^2\sqrt{16-x^2}}$
   (b) $\int \frac{dx}{x^3\sqrt{9x^2-4}}$
   (c) $\int_0^{\pi/2} \frac{\cos t}{\sqrt{1+\sin^2 t}}dt$
   (d) $\int x\sqrt{x^4+1}dx$
3. True/False. For those that are true, prove it. For those that are false, give a counterexample.
   (a) $\sin^{-1}(\sin x) = x$ for all x
   (b) $\sqrt{\sec^2 x - 1} = \tan x$
   (c) $\sin(\sin^{-1} x) = x$ for all x in the domain of $\sin^{-1}$

Completing the Square

1. $\int \frac{dt}{\sqrt{t^2-6t+13}}$
2. $\int \frac{x+3}{\sqrt{2x-x^2}}dx$
3. Sketch the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and find its area. What happens if you plug in $a = b = r$ into your formula? Does this make sense?

Extra Problems If you finish early, take a stab at these.

1. $\int \frac{\cos x - 1}{\cos x + 1}dx$
2. $\int \frac{\sin x + \cos x}{\sin 2x}dx$
3. $\int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}$
4. Let S be a sphere of radius 1. Now take a plane a distance $d < 1$ away from the center and slice the sphere into two pieces. What is the volume of each piece? Hint: the formula for the volume of a revolved solid is $\int_a^b \pi f(x)^2$