The below problems should give you a feel for the flavor of things that might be on the midterm, though obviously there are **way** more problems here than will be on the actual midterm. As always with these sorts of things, the inclusion or exclusion of certain topics from this list **should not be thought of as an indication of the contents of the actual exam**.

Basic Skills & Computations These problems are direct applications of techniques we've been doing. You should be able to find examples & homework problems that are very similar in nature to these.

1. A piece of wire 10cm long is cut into two pieces. The first is bent into an equilateral triangle, and the second is bent into a circle. How big should each of the pieces be so as to maximize the total area enclosed by the two shapes?

2. A rectangular box without a lid is to be made from a piece of cardboard that measures 10cm $\times$ 12cm by cutting out a small square from each corner. Find an expression for the total volume of such a box in terms of the dimensions of the cut-out square. (see 4.4 # 45a for a picture if you aren't understanding the setup)

3. A certain square pyramid is to have a total volume of 36cm^3. Express the lateral surface area as a function of the length of the sides of the base. Hint: for a square pyramid, $V = \frac{1}{3}l^2h$ and $S = l\sqrt{l^2 + (2h)^2}$.

4. A line with positive slope m passes through the point (a, b) which lies in the second quadrant. Express the area of the triangle enclosed by the line and the axes as a function of m

5. What point on the curve $y = \sqrt{x-3} + 3$ is closest to the point $(8, 3)$?

6. A farmer wants to build a rectangular pen along a river. If the river can act as one side of the pen, what is the largest possible area he can enclose with 500ft of fencing?

7. A certain company produces widgets at a cost of \$4 per widget. From market research, the company knows that at a price of p dollars, they can sell $2000 - 100p$ widgets. What price should they set so as to maximize their profit?

8. Graph each of the following functions. Be sure to label all important features of the graph.

   (a) $\dfrac{(x-3)(2x+4)}{x^2-5x}$ (b) $\dfrac{(x-1)(3x+9)}{(x+3)(x+1)}$ (c) $\dfrac{x(x+4)}{(x-2)^2(x+1)}$

9. Simplify each of the following so that only one "log" appears in your answer:

   (a) $\ln(x-3) - 2(\ln(3x+4) + \frac{1}{3}\ln(x^2-4x))$
   (b) $\log_4(3x+1) - 2\log_8(3x+1)$
   Hint: convert to base-2 logs

10. For each of the following functions, find (i) the domain of the function and (ii) the inverse of the function, if it exists

    (a) $3^{1+\frac{1}{x}} - 5$
    (b) $\ln(x-5) - \ln(2x+3)$
    (c) $\ln\ln\ln x$
    (d) $e^{x^2+4}e^{x^2-1}$
    (e) $\ln(x + \sqrt{x^2+1})$

11. Solve each of the following equation or inequalities

    (a) $\log_2(x+2) = \dfrac{1}{2}\log_2 x$
    (b) $e^{6x} - 12e^{3x} = 9$
    (c) $2^{x^2-x} > 32$
    (d) $\log_2(2x^2+4) = 5$
    (e) $(\log_3(x-1))^3 = \log_3(x-1)$
    (f) $\ln\left(\dfrac{3x-2}{4x+1}\right) > \ln 4$
    (g) $\log_6 x = \dfrac{1}{\frac{1}{\log_2 x} + \frac{1}{\log_3 x}}$
    (h) $\log_2 x = \log_x 3$
    (i) $4^x + 2^{x+1} = 48$

12. Suppose θ is an acute angle such that $\sin\theta = \frac{3}{4}$. Find $\cos\theta, \tan\theta, \sec\theta, \csc\theta, \cot\theta$

13. Simplify each of the following as much as possible:

    (a) $\csc\theta\tan^2\theta(1-\cos^2\theta)$

(b) $\cot\theta + \dfrac{1-2\cos^2\theta}{\sin\theta\cos\theta}$

(c) $\dfrac{\cos\theta\tan\theta}{\tan(90^\circ-\theta)} - \dfrac{1}{\sin(90^\circ-\theta)}$

14. Suppose a regular octagon is inscribed in a circle of radius r.

    (a) Find an expression for the area of this octagon in terms of the radius

    (b) Find an expression for the area of this octagon in terms of the length of one of its sides

15. Excercises 25, 34, 38 42 from section 6.3

16. Prove that each of the following are identities:

    (a) $\tan A\tan B = \dfrac{\tan A+\tan B}{\cot A+\cot B}$

    (b) $\csc\theta - \sin\theta = \cot\theta\cos\theta$

    (c) $\dfrac{\sec\theta-\csc\theta}{\sec\theta+\csc\theta} = \dfrac{\tan\theta-1}{\tan\theta+1}$

A Moment's Thought These problems, while related to what we've been doing, are not direct computations and might require a bit of thinking to solve. Working through these problems will hopefully check how well you're understanding the ideas and concepts we've been covering.

1. Find the largest possible area for a rectangle inscribed in the triangle formed by the line $y = mx + b$ and the coordinate axes. You may assume $m < 0$ and $b > 0$.

2. At what point(s) does the graph of $y = \frac{2x^2-3x-2}{x^2-3x-4}$ cross its horizontal asymptote?

3. Find all solutions to $x^{(x^x)} = (x^x)^x$

4. Find the perimeter of a regular pentagon inscribed in a circle of radius 1.

5. Prove the change-of-base formula for logs. You may use the other properties of logs and any of the exponential properties you want.