Math 32 Midterm 2

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You have until 9:30am to complete this test. No calculators, books, notes, or consultation with other members of the class are permitted. Your exam should have 11 pages.

Unsupported or improperly supported answers will receive no credit.

Name: ____________________

GSI: ____________________

Section time: ____________________

Question	Points	Score
1	15	
2	15	
3	10	
4	10	
5	10	
6	15	
7	10	
8	15	
Total:	100	

1. Short answer. You need not show any work for this problem

(a) (6 points) Complete the following table.

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$30°$			
$-120°$			
$135°$			

Solution:

θ	$\sin\theta$	$\cos\theta$	$\tan\theta$
$30°$	$1/2$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
$-120°$	$-\frac{\sqrt{3}}{2}$	$-1/2$	$\sqrt{3}$
$135°$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1

(b) (3 points) "Simplify" so that only one ln appears: $2\ln(x-3) - \frac{1}{2}(\ln(x+1) + \ln x)$

Solution:

$$\ln\left(\frac{(x-3)^2}{\sqrt{x(x+1)}}\right)$$

(This is basically review problem 9a)

(c) (2 points) Evaluate $\log_8 16$ (Hint: Change base)

Solution:

$$\log_8 16 = \frac{\log_2 16}{\log_2 8} = \frac{4}{3}$$

(This is a much simpler version of review problem 9b)

(d) (2 points) Evaluate $e^{\ln 7 - 3\ln 2}$

Solution:

$$e^{\ln\frac{7}{8}} = \frac{7}{8}$$

(e) (2 points) Evaluate $\log_2 48 + 2\log_4(8/3)$

Solution:

$$= \log_2(48) + 2\frac{\log_2(8/3)}{\log_2 4} = \log_2(48 \cdot 8/3) = \log_2(16 \cdot 8) = 4 + 3 = 7$$

(This is basically review problem 9b)

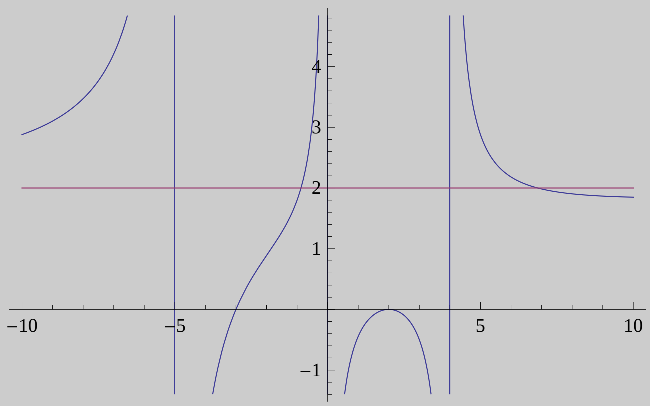
2. (15 points) Sketch a graph of the function $f(x) = \dfrac{(x-2)^2(2x+6)}{x(x+5)(x-4)}$.
You must clearly label all asymptotes and intercepts, including places where the graph crosses an asymptote, if any.

Solution: A quick look shows that the x-intercepts are 2, -3, the vertical asymptotes are 0, 4, -5, and since the degree of the numerator and the degree of the denominator are both 3, we see that the horizontal asymptote is 2. Note that there is no y-intercept because $f(0)$ isn't defined.

We still need to see where this crosses the asymptote $y = 2$:

$$\begin{aligned} \frac{(x-2)^2(2x+6)}{x(x+5)(x-4)} &= 2 \\ (x^2-4x+4)(2x+6) &= 2x(x+5)(x-4) \\ 2x^3-8x^2+8x+6x^2-24x+24 &= 2x(x^2+x-20) \\ 2x^3-2x^2-16x+24 &= 2x^3+2x^2-40x \\ -4x^2+24x+24 &= 0 \\ x^2-6x-6 &= 0 \\ x = \frac{6 \pm \sqrt{36+24}}{2} &= 3 \pm \sqrt{15} \end{aligned}$$

Combining all this gives:



(Note: this is essentially review problems 8a,b,c just with different numbers)

3. (10 points) Find all real solutions to

$$\frac{\ln(\sqrt{x+4}+2)}{\ln\sqrt{x}} = 2$$

Solution:

$$\begin{aligned}
\ln(\sqrt{x+4}+2) &= 2\ln\sqrt{x} \\
\ln(\sqrt{x+4}+2) &= \ln x \quad \text{(log laws)} \\
\sqrt{x+4}+2 &= x \quad \text{(e both sides)} \\
\sqrt{x+4} &= x-2 \\
x+4 &= x^2-4x+4 \quad \text{(Note: check work!)} \\
0 &= x^2-5x = x(x-5)
\end{aligned}$$

So $x = 0, 5$ are our initial guesses. However, $x = 0$ is not in the domain of $\ln\sqrt{x}$, so it cannot be a solution. Checking $x = 5$ gives:

$$\frac{\ln(\sqrt{5+4}+2)}{\ln\sqrt{5}} = \frac{\ln 5}{\frac{1}{2}\ln 5} = 2$$

So $x = 5$ is the only solution.

4. (10 points) Find all values of x for which

$$\log_2 x + \log_2(x+1) - \log_2(x+3) < 1$$

Solution:

$$\begin{aligned}
\log_2\left(x(x+1)\right) &< 1 + \log_2(x+3) \\
2^{\log_2(x(x+1))} &< 2^{1+\log_2(x+3)} = 2^1 2^{\log_2(x+3)} \\
x(x+1) &< 2(x+3) \\
x^2 + x - 2x - 6 &< 0 \\
(x-3)(x+2) &< 0
\end{aligned}$$

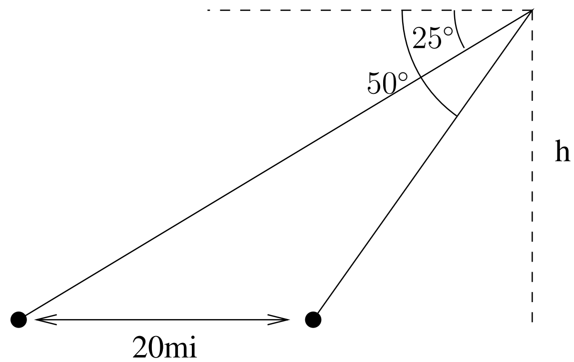
So the key numbers are 3, -2 and a quick check shows that $(x-3)(x+2) < 0$ in $(-2, 3)$

Checking our domain, we see that we need $x > 0, x+1 > 0, x+3 > 0$, which all combine to be just $x > 0$.

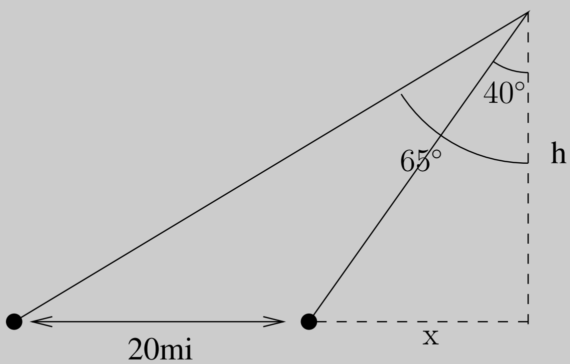
Thus, our final answer is $(0, 3)$

(note: this is a slight re-wording of 5.5 # 69, which was a homework problem)

5. (10 points) While hovering above the ocean, a helicopter pilot notices that his altimeter (the device the measures elevation) is broken. Fortunately for him, he sees two islands directly in front of him that he knows are exactly 20 miles apart. If he must look down at an angle of 50° to see the first one, and at an angle of 25° to see the second, what is his current altitude?



Solution: We start by relabelling a few things:



Then sohcahtoa with the right-most triangle tells us that

$$\tan 40^\circ = \frac{x}{h}$$

Similarly, the big triangle tells us that

$$\tan 65^\circ = \frac{x+20}{h}$$

.

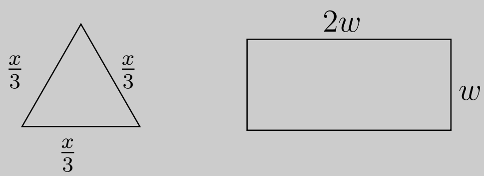
Thus, the first equation tells us that $x = h\tan 40^\circ$, so plugging that into the second gives:

$$\begin{aligned} \tan 65^\circ &= \frac{h\tan 40^\circ + 20}{h} \\ h\tan 65^\circ &= h\tan 40^\circ + 20 \\ h(\tan 65^\circ - \tan 40^\circ) &= 20 \\ h &= \frac{20}{\tan 65^\circ - \tan 40^\circ} \end{aligned}$$

(Note: this is very similar to example 8 from 6.3)

6. (15 points) A piece of wire 12cm long is cut into two pieces, one of length x and the other of length $12-x$. The first is bent into an equilateral triangle and the second is bent into a rectangle whose width is twice its length. What should x be so as to minimize the combined area enclosed by the triangle and the rectangle?

Solution: In pictures, our setup is:



Because the triangle is equilateral, we know that

$$A_{triangle} = \frac{1}{2}ab\sin\theta = \frac{1}{2}\frac{x}{3}\frac{x}{3}\sin 60^\circ = \frac{\sqrt{3}x^2}{36}$$

For the rectangle, we know the perimeter must be $12-x$. Thus, we have $12-x = w+w+2w+2w$. Solving for w gives $w = 2 - \frac{x}{6}$ and thus

$$A_{rect} = w(2w) = 2(2-\frac{x}{6})^2 = 2\left(\frac{x^2}{36} - \frac{4x}{6} + 4\right) = \frac{2}{36}x^2 - \frac{4}{3}x + 8$$

Combining gives:

$$A = A_{triangle} + A_{rect} = \frac{\sqrt{3}x^2}{36} + \frac{2}{36}x^2 - \frac{4}{3}x + 8 = \left(\frac{\sqrt{3}+2}{36}\right)x^2 - \frac{4}{3}x + 8$$

This is minimized at the vertex which is

$$-\frac{b}{2a} = \frac{4/3}{\frac{\sqrt{3}+2}{18}}$$

(Note: this is basically the same as review problem #1)

7. (10 points) Find all real solutions to $9^x - 2 \cdot 3^{x+1} = 27$

Solution:

$$\begin{aligned}
\left(3^2\right)^x - 2 \cdot 3 \cdot 3^x &= 27 \\
3^{2x} - 6 \cdot 3^x - 27 &= 0 \\
t^2 - 6t - 27 &= 0 \ \text{(setting } t = 3^x\text{)} \\
(t-9)(t+3) &= 0
\end{aligned}$$

So $t = 9$ and $t = -3$ are our solutions. Now we solve for x:

$3^x = 9$ has solution $x = 2$

$3^x = -3$ has no solution as $3^{anything}$ is always positive.

Thus, $x = 2$ is the only solution.

(Note: this is review problem 11i with slightly different numbers)

8. (a) (10 points) Prove that $\frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} = 2\csc^2\theta - 2$ is an identity

Solution:

$$\begin{aligned} LHS = \frac{1}{\sec\theta - 1} - \frac{1}{\sec\theta + 1} &= \frac{\sec\theta + 1}{(\sec\theta - 1)(\sec\theta + 1)} - \frac{\sec\theta - 1}{(\sec\theta - 1)(\sec\theta + 1)} \\ &= \frac{\sec\theta + 1 - (\sec\theta - 1)}{\sec^2\theta - 1} \\ &= \frac{2}{\tan^2\theta} \end{aligned}$$

$$\begin{aligned} RHS = 2\csc^2\theta - 2 &= 2\left(\frac{1}{\sin^2\theta} - \frac{\sin^2\theta}{\sin^2\theta}\right) \\ &= 2\frac{1 - \sin^2\theta}{\sin^2\theta} \\ &= 2\frac{\cos^2\theta}{\sin^2\theta} \\ &= \frac{2}{\tan^2\theta} \end{aligned}$$

So LHS = RHS and thus this is an identity.

(Note: This is very similar to example 7 from 6.5. You can also solve it by changing to sin and cos)

(b) (5 points) Suppose θ is an angle such that $-90° < \theta < 0°$. If $\csc\theta = -\frac{4}{3}$, evaluate the remaining five trigonometric functions of θ

Solution:

$\sin\theta = \frac{1}{\csc\theta} = -\frac{3}{4}$

$\cos\theta = \pm\sqrt{1 - \sin^2\theta} = \pm\sqrt{\frac{16}{16} - \frac{9}{16}} = \frac{\sqrt{7}}{4}$

(because of the bound on θ, we know it lies in the fourth quadrant and thus $\cos\theta > 0$)

$\tan\theta = \frac{\sin\theta}{\cos\theta} = -\frac{3}{\sqrt{7}}$

$\cot\theta = \frac{1}{\tan\theta} = -\frac{\sqrt{7}}{3}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{4}{\sqrt{7}}$

(Note: This is a slight tweak of review problem 12)

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