

The below problems should give you a feel for the flavor of things that might be on the midterm, though obviously there are **way** more problems here than will be on the actual midterm. As always with these sorts of things, the inclusion or exclusion of certain topics from this list **should not be thought of as an indication of the contents of the actual exam**.

**Basic Skills & Computations** These problems are direct applications of techniques we've been doing. You should be able to find examples & homework problems that are very similar in nature to these.

1. For each of the following pairs of points, find i) the distance between them, ii) their midpoint, iii) an equation for the line between them.

- (a)  $(1, 3)$  and  $(-2, 7)$   
(b)  $(-3, -1)$  and  $(1, 7)$

2. Find all real solutions to each of the following equations.

- (a)  $x^2 + 3x = 4$   
(b)  $x^2 + x - 4 = 0$   
(c)  $3x^2 - 2x + 1 = 0$   
(d)  $\sqrt{x-3} + \sqrt{2x+4} = 6$   
(e)  $|x-3| + \sqrt{2x-1} = 4$   
(f)  $x^5 + 8x^3 - 5x = 0$   
(g)  $\frac{2}{x^2} + \frac{1}{x} - 3 = 0$   
(h)  $\frac{x^2-x-6}{x+2} = 1$   
(i)  $\frac{1}{x-4} + \frac{3}{x-2} = 0$   
(j)  $(x+3)(x-5) = x+3$

3. Solve each of the following inequalities. Express your answer in interval notation.

- (a)  $\frac{(x-3)(x+2)}{(x+1)^2(x+5)} < 0$   
(b)  $\frac{(x-1)(x+3)}{x-2} \geq 0$   
(c)  $|x-3| + |2x+5| > 0$   
(d)  $\frac{x^3+2x^2-3x}{x+3} \leq 0$   
(e)  $\frac{1}{x} + \frac{1}{x+1} + \frac{1}{x+2} \geq 0$

4. Sketch a graph of each of the following equations/functions. Be sure to clearly label all x and y intercepts as well as any important features.

- (a)  $y = 3x + 4$   
(b)  $x^2 + 6x + y^2 - 8y = 0$   
(c)  $f(x) = \sqrt{x-3} + 2$   
(d)  $f(x) = (2-x)^3 + 1$   
(e)  $f(x) = 3 - |x-2|$   
(f)  $f(x) = 1 - \frac{1}{2-x}$

5. For what values of  $k$  will  $x^2 + kx + k^2 = 0$  have two real roots?

6. Find the domain of each of the following functions. You may write your answer in any mathematically correct way.

- (a)  $f(x) = \frac{1}{x^2+x-6}$   
(b)  $f(x) = \sqrt{\frac{x-2}{x+3}}$   
(c)  $f(x) = \frac{3}{\sqrt{(x+4)(x+1)}}$

7. Find the domain and range of each of the following functions.

- (a)  $f(x) = \frac{x-3}{x+4}$

(b)  $f(x) = \frac{x^2+1}{x^2-4}$

8. Find the average rate of change of  $f(x) = \frac{1}{x+2}$  on  $[2, 5]$

9. Let  $f(x) = x^2 + 3$ . Simplify each of the following:

(a)  $\frac{f(x+h)-f(x)}{h}$

(b)  $\frac{f(x)-f(a)}{x-a}$

10. Same as above, but with  $f(x) = \frac{1}{x}$

11. Find the inverse for each of the following functions, or explain why no inverse can exist

(a)  $f(x) = 3x + 4$

(b)  $f(x) = \frac{x-2}{x+5}$

(c)  $f(x) = \sqrt{x+4}$

(d)  $f(x) = (x-1)^3 - 5$

(e)  $f(x) = (x-4)^2 + 3$

**A Moment's Thought** These problems, while related to what we've been doing, are not direct computations and might require a bit of thinking to solve. Working through these problems will hopefully check how well you're understanding the ideas and concepts we've been covering.

1. Complete the following table. Note that there are many possible answers

$f(x)$	$g(x)$	$h(x)$	$(f \circ g \circ h)(x)$
$x^2$	$x - 3$	$\frac{1}{x}$	
$1/x$		$(12 - x^2)$	$(12 - x^2)^{-2}$
	$\sqrt{x}$		$ \sqrt{x-2} $
			$[1 - (3x - 2)^3]^4$

2. (a) Find the average rate of change for a line with slope  $m$ .

(b) Show that if  $f(x)$  is a function whose average rate of change is  $m$  on every interval, then  $f$  is in fact a line with slope  $m$  (Note: this is a rather tricky problem and is trickier than anything that will be on the actual midterm.)

3. Suppose  $f, g$  are functions such that  $g(x) = x^3 + 2x^2 - 8x$  and  $f^{-1}(x) = \frac{x-1}{x+5}$ . Find all solutions to  $(g \circ f)(x) = 0$

4. A graph of the function  $f(x)$  is show below. Sketch a graph of each of the following, and label the exact coordinates of the points corresponding to the points labeled on  $f$ .

(a)  $f(3 - x)$

(b)  $|f(x + 2)|$

(c)  $8 - f(x)$

(d)  $f(|x|) + 3$

5. (a) Find an equation for the line perpendicular to  $y = 2x + 3$  that passes through the point  $(-1, 5)$ . Express your answer in slope-intercept form.

(b) Let  $P = (5, 0), Q = (1, 8), R = (2, 1), S = (6, 3)$ . Show that the line segments  $\overline{PQ}$  and  $\overline{RS}$  are perpendicular.