Math 32 Midterm 1 SOLUTIONS

Rob Bayer

September 29, 2009

You have until 9:30am to complete this test. No calculators, books, notes, or consultation with other members of the class are permitted. Your exam should have 4 pages.

Unsupported or improperly supported answers will receive no credit.

1. (10 pts) Find all real solutions to $x^6 - 5x^4 = 36x^2$

$$\begin{aligned} x^6 - 5x^4 - 36x^2 &= 0 \\ x^2(x^4 - 5x^2 - 36) &= 0 \end{aligned}$$

By the Zero Product property, either $x^2 = 0$ or $x^4 - 5x^2 - 36 = 0$. To solve the latter, we make the substitution $t = x^2$ and get

$$\begin{aligned} t^2 - 5t - 36 &= 0 \\ (t-9)(t+4) &= 0 \\ t &= 9, -4 \\ x^2 &= 9 \Rightarrow x = \pm 3 \\ x^2 &= -4 \text{ has no real solutions} \end{aligned}$$

Thus, the final answer is just $\boxed{x = 0, 3, -3}$

2. (10 pts) Find all real solutions to $x = \sqrt{3 - 2x} - 6$

$$\begin{aligned} x + 6 &= \sqrt{3 - 2x} \\ (x+6)^2 &= 3 - 2x \quad \text{(*Note: check work since we squared both sides)} \\ x^2 + 12x + 36 &= 3 - 2x \\ x^2 + 14x + 33 &= 0 \\ (x+3)(x+11) &= 0 \end{aligned}$$

So $x = -3, -11$ are potential solutions. By our previous note, we should check our work:

$\underline{x = -3}$: $-3 \stackrel{?}{=} \sqrt{3 - 2(-3)} - 6 = \sqrt{9} - 6 = 3 - 6 = -3$, so $x = -3$ is a solution

$\underline{x = -11}$: $-11 \stackrel{?}{=} \sqrt{3 - 2(-11)} - 6 = \sqrt{25} - 6 = -1$, so $x = -11$ is NOT a solution.

So our final answer is $\boxed{x = -3}$

3. (10 pts) Solve the inequality $|4 - 3x| > 8$. Express your answer using interval notation.

We know this is equivalent to $4 - 3x > 8$ or $4 - 3x < -8$. So we get:

$$\begin{aligned} 4 - 3x &> 8 \\ -3x &> 4 \\ x &< -\frac{4}{3} \end{aligned}$$

or

$$\begin{aligned} 4 - 3x &< -8 \\ -3x &< -12 \\ x &> 4 \end{aligned}$$

So our final answer is $\boxed{(-\infty, -\frac{4}{3}) \cup (4, \infty)}$

4. (10 pts) Find an equation for the line that contains the point $(-1, 3)$ and is perpendicular to the line $3x+2y = -6$. Express your answer in the form $y = mx + b$

   Let's start be re-writing the given line equation as $y = -\frac{3}{2}x - 3$. This shows us that the slope of this line is $\frac{-3}{2}$ so the slope of the perpendicular line must be $\frac{2}{3}$. Using the point-slope formula for a line, we get

$$\begin{aligned} y - 3 &= \frac{2}{3}(x + 1) \\ y &= \frac{2}{3}x + \frac{2}{3} + 3 \\ y &= \frac{2}{3}x + \frac{11}{3} \end{aligned}$$

   which is of the required form

5. (15 pts) Solve the inequality $\dfrac{x^2 - 3x}{x^2 + 5x + 4} \geq 0$. Express your answer in interval notation.

   We start by factoring the top and bottom to get $\dfrac{x(x-3)}{(x+4)(x+1)}$

   The key numbers here are $-4, -1, 0, 3$. We now do the standard trick of drawing a number line and looking at what happens in each interval:

	$x < -4$	$-4 < x < 1$	$-1 < x < 0$	$0 < x < 3$	$x < 3$
x	-	-	-	+	+
$x - 3$	-	-	-	-	+
$x + 4$	-	+	+	+	+
$x + 1$	-	-	+	+	+
$\frac{x(x-3)}{(x+4)(x+1)}$	+	-	+	-	+

   Since $x = 0$ and $x = 3$ both give us 0, our overall answer is

   $\boxed{(-\infty, -4) \cup (-1, 0] \cup [3, \infty)}$

6. (15 pts) Let $f(x) = \frac{2x^3+3}{x^3+8}$.

   (a) (9 pts) Find $f^{-1}(x)$ or show that f does not have an inverse

   We will interchange y and x and solve for y:

$$
\begin{aligned}
x &= \frac{2y^3+3}{y^3+8} \\
x(y^3+8) &= 2y^3+3 \\
xy^3+8x &= 2y^3+3 \\
xy^3-2y^3 &= 3-8x \\
(x-2)y^3 &= 3-8x \\
y^3 &= \frac{3-8x}{x-2} \\
y &= \sqrt[3]{\frac{3-8x}{x-2}}
\end{aligned}
$$

So $\boxed{f^{-1}(x) = \sqrt[3]{\frac{3-8x}{x-2}}}$

(b) (6 pts) Determine the domain and range of f.
Hint: when finding the range, feel free to reference your work from part (a)

The only possible problem with f would be if $x^3+8=0$. That is, if $x^3=-8$. ie, if $x=\sqrt[3]{-8}=-2$. Thus, the domain of f is all x's except -2

For the range, we just need to find the domain of f^{-1}. Again, the only problem here is if $x-2=0$, which would happen if $x=2$. Thus, the range is all x's except 2.

7. (10 pts) For what values of k will the equation $x^2+2kx+k=0$ have no real solutions?

For there to be no real solutions, the discriminant would have to be negative. Ie, $(2k)^2-4k<0$. Factoring this give $4k(k-1)<0$, so after dividing through by 4 we see that we really just need to solve $k(k-1)<0$. The key numbers here are $0, 1$ and we get:

	$k<0$	$0<k<1$	$1<k$
k	-	+	+
$k-1$	-	-	+
$k(k-1)$	+	-	+

So the final answer is $\boxed{0<k<1}$

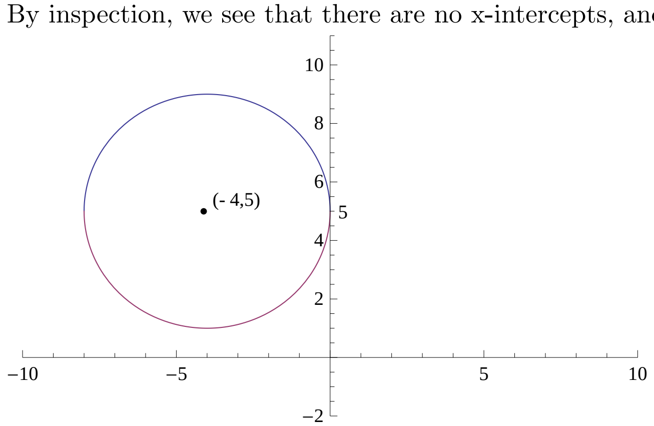
8. (20 pts) Sketch a graph of each of the following equations/functions.

Be sure to **clearly label** the coordinates of any x- or y- intercepts as well as any important features of the graph. Note that this problem continues onto the next page

(a) (10 pts) $x^2+y^2+8x-10y+25=0$
We complete the square and get:

$$(x+4)^2-16+(y-5)^2-25+25=0$$

which can be re-written as $(x+4)^2+(y-5)^2=16$. Thus, this is a circle of radius 4 centered at $(-4,5)$. By inspection, we see that there are no x-intercepts, and that the y-intercept is at 5. Thus, we have:



(b) (10 pts) $f(x) = 2 - \sqrt{1-x}$ (Remember to label all important features, including any x or y intercepts)

To find the y-intercepts, we plug in $x = 0$ to get $y = 2 - \sqrt{1} = 1$.

For the x-intercepts, we solve $2 - \sqrt{1-x} = 0$, so $2 = \sqrt{1-x}$, so $x = -3$.

Thus, after performing appropriate translations and reflections we get