You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.
Feel free to skip around and spend your time on things your group feels sketchy about. Also, you should focus not just on the answers to each problem, but also on how you would actually write up the solution on the midterm.

Chain Rule

1. Complete the following table. Note that there are many possible answers

$f(x)$	$g(x)$	$h(x)$	$(f \circ g \circ h)(x)$
x^2	$\sin x$	$\cos 4x$	
$1/x$		$(12 - x^2)$	$(12 - x^2)^{-2}$
	x		$\cos \tan x$
			$[1 - (3x - 2)^3]^4$

2. Find $\frac{d}{dx}$ for each of the following. Don't bother simplifying.
   (a) $\sin x^2$
   (b) $\sin^2 x$
   (c) $\sin(\sin x)$
   (d) $\sqrt[3]{2x + 4} + \frac{1}{\cos 2x} - x \sin(\cos x)$
   (e) $\sin(\cos(\tan x))$
   (f) $\sqrt{1 + 2 \sin 3x^2}$

3. True/False? For those that are true explain why. For those that are false, correct the sentence.
   (a) $(fg)' = f'g'$
   (b) $(f + g)' = f' + g'$
   (c) $(f/g)' = f'/g'$
   (d) $(f - g)' = f' - g'$
   (e) $(f \circ g)' = f' \circ g'$

4. Use the chain rule to show that
   (a) The derivative of an odd function is even
   (b) The derivative of an even function is odd

   Hint: consider $\frac{d}{dx}(f(-x))$

5. Suppose on the next midterm you completely blank on the quotient rule. Show that you can come up with the quotient rule using only the product rule, the chain rule, and the power rule.

6. What is $(f \circ g)''$?

Implicit Differentiation

1. For each of the following curves, find $\frac{dy}{dx}$, assuming y is a function of x. Your answer will probably involve both y and x.
   (a) $5x^2y + xy^2 = 3x$
   (b) $1 + x = \sin(xy^2)$
   (c) $\sqrt{xy} = 1 + \cos x$
   (d) $\sin x + \cos y = \sin x \cos y$

2. Find y'' in terms of x, y, y' for the curve defined by $\sqrt{x} + \sqrt{y} = 1$

3. Find the equation for the tangent line to the given curves at the given points:

   (a) $2(x^2 + y^2)^2 = 25(x^2 - y^2)$, at $(3, 1)$

   (b) $x^{2/3} + y^{2/3} = 4$, at $(-3\sqrt{3}, 1)$

4. Consider the curve defined by the formula $y^3 + xy + x^4 = 11$.

   (a) Find y'' at the point $(1, 2)$.

   (b) Find a general formula for $\frac{d^2y}{dx^2}$ that involves only x and y (ie, $\frac{dy}{dx}$ should not appear in your formula)

5. (a) Geometrically, what does it mean for two line to be perpendicular? Algebraically, how can you tell?

   (b) Geometrically, what does it mean for two curves to be perpendicular at a given point? In terms of their derivatives at that point, how can you tell?

   (c) Two families of curves are said to be orthogonal if they are perpendicular everywhere they intersect. Show that the family[1] of curves $x^2 + y^2 = r^2$ is orthogonal to the family $y = kx$. Does this make sense geometrically?

   (d) Now do the same for the families $x^2 + y^2 = ax$ and $x^2 + y^2 = by$.

6. Consider the equation $x^2 + y^2 = 5$, and suppose both $x = x(t)$ and $y = y(t)$ are functions of the independent variable t. Find $\frac{dy}{dt}$ in terms of x, y, and $\frac{dx}{dt}$.

[1] We call $x^2 + y^2 = r^2$ a family of curves because each choice of r gives a new curve