You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.
Feel free to skip around and spend your time on things your group feels sketchy about. Also, you should focus not just on the answers to each problem, but also on how you would actually write up the solution on the midterm.

Calculations

1. Find each of the following limits, if they exist:

   (a) $\lim_{x\to\infty} \sqrt{4x^4+x} - 2x^2$

   (b) $\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$

   (c) $\lim_{t\to 0} \left(\dfrac{1}{t} - \dfrac{1}{t^2+1}\right)$

   (d) $\lim_{h\to 0} \dfrac{\sqrt{1+h}-1}{h}$

   (e) $\lim_{x\to\infty} (x - \sqrt{x})$

   (f) $\lim_{x\to\infty} \dfrac{\cos^2 x}{x^2}$

   (g) $\lim_{t\to 0} \dfrac{\sin^2 3t}{t^2}$

   (h) $\lim_{x\to -3} \dfrac{x+2}{x+3}$

   (i) $\lim_{x\to -3^+} \dfrac{x+2}{x+3}$

   (j) $\lim_{x\to 0} x^4 \cos\left(\dfrac{2}{x}\right)$

2. True/False. For those that are true, explain why. For those that are false, provide a counterexample.

   (a) If $\lim_{x\to a} f(x) + g(x)$ exists and is finite, then $\lim_{x\to a} g(x)$ also exists and is finite.

   (b) If $\lim_{x\to a} f(x)$ and $\lim_{x\to a} f(x) + g(x)$ both exist and are finite, then $\lim_{x\to a} g(x)$ exists and is finite.

   (c) If $\lim_{x\to a} f(x)g(x)$ exists, then $\lim_{x\to a} f(x)$ does too.

3. Find all horizontal and vertical asymptotes of $y = \dfrac{2x^2+x-1}{x^2+x-2}$.

4. Find the domain and range and sketch a graph of $y = \dfrac{1}{\sqrt{4-x^2}-2}$.

5. Find all horizontal asymptotes of $y = \dfrac{x^2 + x\sin x}{\sqrt{x^4+x}}$

6. Show that the function $f(x) = \begin{cases} x\sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$ is continuous at 0, but is not differentiable there.

7. Find the derivative of each of the following. Don't bother simplifying.

   (a) $\dfrac{t^2}{3t^2-2t+1}$

   (b) $\sin\theta + \frac{1}{2}\cot\theta$

   (c) $(\sqrt{x} + \sin x)\cos x$

   (d) $\dfrac{x^2+\tan x}{\sqrt{x}+x}$

8. Find the equation for the line tangent to $y = \frac{2x}{x+1}$ at $(1,1)$

9. Find $\frac{d^3 5}{dx^3 5}(x\sin x)$

10. Suppose n is a positive number. Find $\frac{d}{dx}\frac{1}{x^n}$ using (a) the quotient rule, (b) the power rule with a negative exponent. Do your answers agree?

11. Consider the circle $(x-2)^2 + (y-3)^2 = 1$. How many tangent lines to this circle pass through the origin? Find an equation for each of them.

Proofs and Definitions

1. Use the definition of derivative to find $f'(a)$ for each of the following choices of f and a:

   (a) $f(x) = x^2, a = 3$

   (b) $f(x) = \sin x, a = \pi/3$

   (c) $f(x) = 1 - x^3, a = 0$

2. Use the definition of a limit to prove each of the following are true:

(a) $\lim_{x\to 2} x^2 + x - 1 = 5$

(b) $\lim_{x\to 5} \sqrt{x-1} = 2$

(c) $\lim_{x\to\infty} \dfrac{x^4+2}{x^4+3} = 1$

(d) $\lim_{x\to 2} \dfrac{1}{(x-2)^2} = \infty$

(e) $\lim_{x\to\infty} \dfrac{\sqrt{x}+1}{\sqrt{x}-1} = 1$

(f) $\lim_{x\to 4^+} \dfrac{1}{\sqrt{x-4}} = \infty$

3. Prove that each of the following equations has at least one solution:

(a) $x^2 + \sin x = 3$

(b) $\tan x = 2x$

(c) $\sqrt{x-5} = \frac{1}{x+3}$