You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Basic Rules

1. Differentiate each of the following:
   (a) $x^3 - 2x^2 + 3$
   (b) $2x^{1/3}$
   (c) $\sin t - 3\cos t$
   (d) $\sqrt{x+1}(x+1)$
   (e) $\dfrac{x+3\sqrt{x}}{\sqrt[3]{x^4}}$
2. Show that the curve $y = 6x^3 + 5x - 3$ has no tangent with slope 4
3. The equation for the height of an object falling with no air resistance is given by $h(t) = -\frac{g}{2}t^2 + v_0 t + h_0$.
   (a) Find equations for the velocity and acceleration of the object at time t.
   (b) Why are h_0 and v_0 appropriate names?
4. The equation for the volume of a sphere of radius r is $V(r) = \frac{4}{3}\pi r^3$. What is $V'(r)$? Does this formula look familiar? Why does this make sense?
5. A tangent line is drawn to the hyperbola $xy = c$ at a point P (call it $(x, \frac{c}{x})$ if you prefer). Draw a picture to show that the coordinate axes define a line segment of this tangent line.
   (a) Show that regardless of the choice of P, the area enclosed by the coordinate axes and the tangent line is always the same.
   (b) Show that regardless of the choice of P, P is always the midpoint of the line segment defined by the coordinate axes.
6. Let $f(x) = \sin x$. By finding a pattern in the first few derivatives, find $f^{(99)}(x), f^{(100)}(x), f^{(101)}(x)$. Recall that this notation means the 99th, 100th, and 101st derivatives of f, respectively.
7. By finding a pattern in the first few derivatives, find a general formula for $\dfrac{d^n}{dx^n}\dfrac{1}{x}$. The factorial function $(m! = m(m-1)(m-2)\cdots 3\cdot 2\cdot 1)$ may be helpful here.

Product and Quotient Rules

1. Find the derivative of each of the following:
   (a) $x\sin x$
   (b) $\tan x$
   (c) $\dfrac{3x-1}{2x+1}$
   (d) $\sin(2x)$ (do NOT use the chain rule)
   (e) $\dfrac{\sin x}{x}$
   (f) $(x+1)(x^2+1)$
2. Find all points on $y = \dfrac{\cos x}{2+\sin x}$ with a horizontal tangent line
3. We said earlier that $\frac{d}{dx}cf(x) = c\frac{d}{dx}f(x)$. Prove this using the product rule.
4. Calculate $\frac{d}{dx}\frac{1}{x}$ using (a) the power rule, (b) the quotient rule. Do you get the same thing?
5. Prove the product rule using the $h \to 0$ definition of derivative. It might be helpful to add and subtract $f(x+h)g(x)$ from the numerator.
6. How many lines tangent to the curve $y = \frac{x}{x-1}$ pass through the point $(1,2)$? At which points do they touch the curve?