You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

The Definition of Derivative

1. Evaluate each of the following limits. In terms of derivatives, what do they represent?

   (a) $\lim_{x\to 4} \dfrac{x^2-16}{x-4}$ (b) $\lim_{x\to a} \dfrac{x^2-a^2}{x-a}$ (c) $\lim_{h\to 0} \dfrac{(x+h)^2-x^2}{h}$

2. For each of the following functions, (i) graph the function, (ii) sketch a few secant lines through the given point, (iii) sketch the tangent line at the given point, and (iv) find an equation for the tangent line. Do NOT use any differentiation rules other than the definition of derivative.

   (a) $\frac{1}{x}, x = 2$

   (b) $\sin x, x = 0$

3. Using the definition of derivative, find $f'(a)$ for each of the following choices of $f(x)$

   (a) $t^4 - 5t$

   (b) $\frac{x^2+1}{x-2}$

   (c) $\frac{1}{\sqrt{x+2}}$

4. Prof. Hald showed in class that every differentiable function is continuous. To show that the converse is not true, give an example of a function that is continuous at some point a but is not differentiable there.

The Derivative as a Function

1. Which of the following expressions represent numbers and which represent functions? For all problems, a represents an arbitrary constant.

   (a) $f'(a)$

   (b) $Df(x)$

   (c) $\lim_{x\to a} \dfrac{f(x)-f(a)}{x-a}$

   (d) $\left.\dfrac{df}{dx}\right|_{x=a}$

   (e) $f'(x)$

   (f) $\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$

2. For each of the below sets of graphs, decide which is f, f', and f'':

3. The graph of two functions are shown below. Use them to sketch graphs of f', f'', g', g''.

4. Using the definition of derivative and the fact that $\sin(A+B) = \sin A \cos B + \cos A \sin B$, find $\frac{d}{dx} \sin x$

5. True/False. If true, prove it. If false, provide a counterexample. Let $f(x)$ be a function with derivative $f'(x)$. If $\lim_{x \to 0^+} f'(x) = \lim_{x \to 0^-} f'(x)$, then $f'(0)$ exists.

6. Using the $\epsilon - \delta$ definition of a limit, prove that $\dfrac{d}{dx} x^2 = 2x$. Hint: what is L? a?

7. Using limit laws, prove that $\frac{d}{dx} cf(x) = c \frac{d}{dx} f(x)$