You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

The Intermediate Value Theorem

1. State the intermediate value theorem. Be sure to include any and all hypotheses.
2. Prove that $x^5 - x^2 + 2x + 3 = 0$ has at least one real root
3. Prove that $\tan x = 2x$ has at least one solution
4. Prove that $x = \cos x$ has at least one solution
5. Prove that there is at least one number that is exactly 1 more than its cube.

To Infinity! (but sadly not beyond)

1. Sketch the graph of a function with all of the following properties:
   (a) $\lim_{x\to 0} f(x) = \infty$
   (b) $\lim_{x\to 2^+} f(x) = \infty$
   (c) $\lim_{x\to 2^-} f(x) = 1$
   (d) $\lim_{x\to\infty} f(x) = 2$
   (e) $\lim_{x\to -\infty} f(x) = -1$
   (f) f is continuous everywhere except 0 and 2.
2. Find each of the following limits. Note that DNE is no longer a proper response if the limit is actually $\pm\infty$[1]
   (a) $\lim_{x\to\infty} \dfrac{x^4 - x^2 + 1}{x^5 + x^3 - x}$
   (b) $\lim_{x\to 1} \dfrac{3}{x-1}$
   (c) $\lim_{x\to 1^-} \dfrac{3}{x-1}$
   (d) $\lim_{x\to\infty} \dfrac{\sqrt{1+4x^2}}{4+x}$
   (e) $\lim_{x\to 2} \tan\left(\dfrac{2\pi}{x^2}\right)$
   (f) $\lim_{x\to\infty} \sqrt{x^2+1} - \sqrt{x^2-1}$
   (g) $\lim_{x\to\infty} \sin x$
   (h) $\lim_{x\to -\infty} \dfrac{1}{x}\sin x$
   (i) $\lim_{x\to -1} \dfrac{x+2}{(x+1)^2}$
   (j) $\lim_{x\to\infty} \sqrt{x}$
3. Find all horizontal asymptotes of the function $f(x) = \frac{2x^2+x-1}{x^2+x-2}$
4. Let $P(x)$ be a polynomial with leading coefficient a and $Q(x)$ be a polynomial with leading coefficient b. What is $\lim_{x\to\infty} \dfrac{P(x)}{Q(x)}$ if the degree of $P(x)$ is
   (a) strictly less than the degree of $Q(X)$
   (b) strictly greater than the degree of $Q(X)$
   (c) equal to the degree of $Q(X)$

Proofs with ∞

1. Write down the formal definition of $\lim_{x\to\infty} f(x) = L$. What would you guess it should be for $x \to -\infty$?
2. Write down the formal definition of $\lim_{x\to a} f(x) = \infty$ What is it for $= -\infty$?
3. Prove that $\lim_{x\to\infty} \dfrac{1}{(x-2)^2} = 0$
4. Prove that $\lim_{x\to 1} \dfrac{1}{(x-1)^2} = \infty$
5. Prove that $\lim_{x\to -\infty} x^2 = \infty$. Note: part of the problem here is defining what it means to have a limit at infinity be infinity. See if you can figure it out by combining the definitions for $\lim_{x\to\infty}$ and $\lim = \infty$.

[1] That's not to say that DNE is **never** a proper response. Several of these limits do not exist and you should be sure to indicate that.