You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Calculating Limits/Limit Laws

1. Find each of the following limits, or show that they do not exist:

   (a) $\lim_{x\to 1} \frac{x^2+3x-1}{x+4}$

   (b) $\lim_{x\to 2} \frac{x^2+x-6}{x-2}$

   (c) $\lim_{y\to 0} \frac{y^3+2y^2+3y}{y^2+y}$

   (d) $\lim_{t\to \pi/2} t\sin t$

   (e) $\lim_{x\to 4} \frac{x^2-4x}{x^2-3x+4}$

   (f) $\lim_{x\to 0} \frac{x}{\sqrt{1+3x}-1}$

   (g) $\lim_{x\to 0} \frac{\cos x}{2e^x}$

   (h) $\lim_{h\to 0} \frac{(4+h)^2-16}{h}$

   (i) $\lim_{s\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x}$

   (j) $\lim_{x\to -6} \frac{2x+12}{|x+6|}$

2. Use the squeeze theorem to find each of the following limits:

   (a) $\lim_{x\to 0} x\cos\frac{1}{x}$

   (b) $\lim_{x\to 0^+} \sqrt{x}[1+\sin^2(2\pi/x)]$

3. Use the fact that $\lim_{x\to 0}\frac{\sin x}{x}=1$ to find each of the following:

   (a) $\lim_{\theta\to 0} \frac{\sin 5\theta}{\theta}$

   (b) $\lim_{x\to 0} \frac{\sin 2x}{\sin 5x}$

   (c) $\lim_{t\to 0} t\cot 2t$

   (d) $\lim_{x\to 0} \frac{\sin^2 2x}{x^2}$

   (e) $\lim_{x\to 0} \frac{\tan 2x}{\tan 3x}$

   (f) $\lim_{x\to 0} \frac{\sin x^2}{x^2}$

4. Find examples of functions $f(x)$ and $g(x)$ such that neither $\lim_{x\to 0} f(x)$ nor $\lim_{x\to 0} g(x)$ exist but

   (a) $\lim_{x\to 0} f(x)+g(x)$ does

   (b) $\lim_{x\to 0} f(x)g(x)$ does. (This one is kinda tricky–feel free to draw a picture or define a piecewise function)

   (c) $\lim_{x\to 0} f(x)/g(x)$ does

   (d) $\lim_{x\to 0} f(g(x))$ does

5. Prove that $\lim_{x\to 4} x^2 = 16$

6. (Tricky, but definitely doable) Prove that $\lim_{x\to 2}\frac{1}{x^2}=\frac{1}{4}$ using the $\epsilon-\delta$ definition of a limit

Continuity

1. Where are each of the below pictured functions continuous? Classify each discontinuity as a hole or infinite discontinuity.

2. Determine where each of the following functions are continuous:

(a) $\frac{1}{x}$

(b) $\sin x$

(c) $\frac{1}{\sqrt{1-x}}$

(d) $\begin{cases} 1-x^2 & \text{if } x<1 \\ x-2 & \text{if } 1\le x<4 \\ \sqrt{x} & \text{if } x\ge 4 \end{cases}$

3. Show that the function $f(x) = \begin{cases} x^2 & x<1 \\ \sqrt{x} & x\ge 1 \end{cases}$ is continuous at 1

4. Find an example of a function that is neither continuous from the left nor from the right at some point a

5. (Tricky) Consider the function $f(x) = \begin{cases} 1 & \text{if x is irrational} \\ 0 & \text{if x is rational} \end{cases}$

   (a) Explain why this function is not continuous anywhere

   (b) What is $f \circ f$? Where is it continuous?

6. True/False. If f is continuous at a, then so is $f(x)^2$.

7. True/False. If $f(x)^2$ is continuous at a then $f(x)$ is too.

8. Prove, using the $\epsilon - \delta$ definition of a limit, that $f(x) = |x|$ is continuous everywhere.

9. (Tricky) Prove that if f is continuous from both the left and from the right at a, then it is continuous at a

The Intermediate Value Theorem

1. State the intermediate value theorem. Be sure to include **all** hypotheses.

2. Prove that $x^5 - x^2 + 2x + 3 = 0$ has at least one real root

3. Prove that $\tan x = 2x$ has at least one solution

4. Prove that $x = \cos x$ has at least one solution

5. Prove that there is at least one number that is exactly 1 more than its cube.