Math 1A PDP Worksheet

You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

1. Find each of the following limits (or explain why they do not exist), where f is the function whose graph is shown below:
   (a) $\lim_{x\to 0} f(x)$
   (b) $\lim_{x\to 1} f(x)$
   (c) $\lim_{x\to -1} f(x)$
   (d) $\lim_{x\to 3^+} f(x)$
   (e) $\lim_{x\to 3^-} f(x)$
   (f) $\lim_{x\to 3} f(x)$
2. Sketch a graph of a function with the following properties:
   - $\lim_{x\to 0^+} f(x) = 2$
   - $\lim_{x\to 0^-} f(x) = -1$
   - $\lim_{x\to 3} f(x) = 0$
   - $\lim_{x\to -1} f(x) = 4$
   - $f(0) = 0$
   - $f(3) = 1$
3. Let L and a be real numbers. Sketch a graph of a function where:
   (a) $\lim_{x\to a} f(x) = L$ and $f(a) = L$
   (b) $\lim_{x\to a} f(x) = L$ but $f(a) \neq L$
   (c) $\lim_{x\to a^+} f(x)$ and $\lim_{x\to a^-} f(x)$ both exist, but $\lim_{x\to a} f(x)$ does not
4. Suppose δ is some positive real number and a is some real number. Express each of the following sets of numbers
   - Using inequalities
   - Using absolute values
   - Using intervals

   (a) All the numbers that are within δ of 0
   (b) All the numbers that are within δ of a
   (c) All the numbers that are within δ of a but not equal to a

(Somewhat) easy $\epsilon - \delta$ Proofs

1. Write down the formal definition of $\lim_{x\to a} f(x) = L$
2. Prove that $\lim_{x\to 2} 3x - 4 = 2$
3. Prove that $\lim_{x\to -1} \frac{x+1}{2} = 0$
4. Prove that $\lim_{x\to 0} |x| = 0$ (Don't make this harder than it has to be...)
5. Prove that $\lim_{x\to a} c = c$ where a, c are any real numbers.
6. Prove that $\lim_{x\to a} mx + b = ma + b$ where m, a, b are any real numbers

More challenging $\epsilon - \delta$ proofs

1. Let $f(x) = x^2$
   (a) Find all positive numbers x such that x^2 is within 1 of 9
   (b) Using the Min function, find a number δ such that whenever x is within δ of 3, $f(x)$ is within 1 of 9
2. Prove that $\lim_{x\to 3} x^2 + x - 4 = 8$
3. Prove that $\lim_{x\to 4} \sqrt{x} = 2$
4. Prove that $\lim_{x\to 1} \frac{1}{x} = 1$
5. The formal definition of $\lim_{x\to a^-} f(x) = L$ is: $\forall\epsilon\exists\delta$ s.t. $0 < a - x < \delta \Rightarrow |f(x) - L| < \epsilon$
   (a) What is different about this definition vs that of $\lim_{x\to a} f(x) = L$?
   (b) Use this definition to prove that $\lim_{x\to 9^-} \sqrt[4]{9 - x} = 0$