

You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

### More Volumes

- Find the volume of the solid obtained by rotating each of the following regions about the specified axis. You should use cylindrical shells for all these problems.
  - $x = 2\sqrt{y}, x = 0, y = 1$ ; about the x-axis
  - $y = \frac{1}{1+x^2}, y = 0, x = 0, x = 2$ ; about the y-axis
  - $y = x^2, y = 0, x = 1, x = 2$ ; about  $x = 1$
- Consider the solid obtained by rotating the region bounded by  $y = 1 - x^4$  and  $y = 0$  about the line  $x = 2$ . Find the volume of this solid using a) cylindrical shells, b) the washer/disc method.
- On the homework, you will use the washer/disk method to show that the volume of a torus is  $V = 2\pi^2 Rr^2$ . Show that you get the same thing using cylindrical shells.
- Find the volume of the solid obtained by rotating the specified region about the specified axis. You may use any method you prefer. Hint: if you get stuck, try the other method.
  - $y = \cosh x, y = 0, x = 0, x = 1$ ; about the y-axis
  - $y = \ln x, y = 0, x = 1, x = 10$ ; about the x-axis
  - $y = \sec^{-1} x, x = 0, x = 1$ ; about the y-axis
  - $y = 4x - x^2, y = 8x - 2x^2$ ; about  $x = -2$
- What is the volume of material removed by drilling a hole of radius  $r$  through the center of a sphere of radius  $R$ ? You may assume  $R > r$ .

### Integration Practice

If you finish the above problems early, try these as a quick integration review.

$$1. \int \frac{x}{\sqrt{5-x^2}} dx$$

$$2. \int \tan \theta d\theta$$

$$3. \int_1^{10} x^2 \sqrt{x-1} dx$$

$$4. \int dt$$

$$5. \int_0^1 x + \sqrt{1-x^2} dx$$

$$6. \int_{-1}^1 \frac{\sin x}{1+x^2} dx$$

$$7. \int \sinh(1-6x) dx$$

$$8. \int \frac{\sec \theta \tan \theta}{1 + \sec \theta} d\theta$$

$$9. \int \frac{x}{\sqrt{1-x^4}} dx$$