Math 1A PDP Worksheet

Fundamental Theorem of Calculus

1. What is the independent variable in $\int_0^x f(t)dt$?
2. Find each of the following. Be sure to use the proper variable name in your answer:

   (a) $\frac{d}{dx}\int_0^x t\sin^{-1}(t)dt$; $(-1 < x < 1)$

   (b) $\frac{d}{d\theta}\int_{\pi/2}^{\theta} e^{2x}dx$

   (c) $\frac{d}{dx}\int_x^2 \frac{\sin t}{t}dt$

   (d) $\frac{d}{dt}\int_3^{\cos t} e^{x^2}dx$

   (e) $\frac{d}{dx}\int_{2x}^{x^3} \frac{e^t}{t}dt$; $(x > 0)$

3. Why did I put restrictions on x in (a) and (e) above? Why did I not need to do the same for (c)?
4. Find the average value of $f(x) = \sin x$ on the interval $(0, \pi)$.
5. A graph of the function f is drawn at right.
   Let $g(x) = \int_0^x f(t)dt$.
   (a) Where does g have local extrema?
   (b) Where is g concave up? down?
   (c) Where are the absolute extrema?

   Go back and answer (a) and (c) by thinking only about areas

6. (a) Give an example of a continuous function that doesn't have a derivative.
   (b) Prove that every continuous function has an antiderivative. Note: this has a 2-line solution, so don't do anything too crazy.
   (c) It is a (very hard to prove) theorem that $\int e^{x^2}dx$ cannot be written in any nice way. Despite that, write down a formula that gives all antiderivatives of e^{x^2}
7. Prove the Mean Value Theorem for integrals, which states: If $f(t)$ is continuous on $[a, b]$, then there is a c such that $f(c) = \frac{1}{b-a}\int_a^b f(t)dt$. Hint: apply the regular MVT to $F(x) = \int_a^x f(t)dt$
8. Suppose it costs a certain toy company $c(x)$ dollars to produce the xth toy. If it can turn around and sell these toys at a price of P dollars, how many toys should it produce in order to maximize its profit? You may assume $c(x)$ is an increasing function.[1]

u-Substitution

1. Find each of the following:

   (a) $\int xe^{x^2}dx$

   (b) $\int \frac{\cos(\pi/x)}{x^2}dx$

   (c) $\int_1^2 x\sqrt{x-1}dx$

   (d) $\int_0^1 \frac{e^x}{e^x+1}dx$

   (e) $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

   (f) $\int_0^a x\sqrt{a^2-x^2}dx$

2. Prove each of the following:
   (a) If f is even, then $\int_{-a}^a f(x)dx = 2\int_0^a f(x)dx$
   (b) If f is odd, then $\int_{-a}^a f(x)dx = 0$
   (c) Explain geometrically why each of these makes sense.
3. Find $\int(x^3+1)^{-1/2}x^5dx$
4. Find $\int \cos^4 x - \sin^4 x dx$
5. Find $\int_{-1}^1 \frac{\sin x}{1+x^2}dx$

[1] If you know some economics, you should recognize this as "price = marginal cost"