Evaluating Definite Integrals

1. Use the Evaluation Theorem to find each of the following:

   (a) $\int_1^4 \sqrt{t}dt$

   (b) $\int_0^{\pi/4} \sec^2 xdx$

   (c) $\int_0^1 \frac{3}{1+y^2}dy$

   (d) $\int_1^e \frac{x^2+x+1}{x}dx$

   (e) $\int_\pi^{3\pi/2} \cos\theta d\theta$

   (f) $\int_0^1 \frac{x^2}{1+x^2}dx$

2. If it costs a certain toy company $c(x) = 12 + .01x$ dollars to produce the xth toy, what does $\int_0^{100} c(x)$ represent?

3. Suppose $f(x)$ is some function whose value has units *columbs/s*. If x is measured in seconds what are the units of $f'(x)$? What about $\int_a^b f(x)dx$?

4. Find each of the following indefinite integrals:

   (a) $\int e^{x+3}dx$

   (b) $\int \frac{1+\cos^2 t}{\cos^2 t}dt$

   (c) $\int \frac{3x+1}{\sqrt{x}}$

   (d) $\int \frac{6}{\sqrt{1-x^2}}$

5. (a) Evaluate $\int_0^x \sin t + 3t^3 dt$.

   (b) Find $\int \sin x + 3x^3 dx$

   (c) What is different about your answers to parts (a) and (b)?

   (d) In general, what is the difference between $\int_0^x f(t)dt$ and $\int f(x)dx$?

6. A certain paricle moving along a straight line has acceleration $a(t) = t + 4$. If the initial velocity of the particle is $5m/s$, find each of the following:

   (a) The velocity at time t

   (b) The total distance travelled in the first 10 seconds.

7. (a) Evaluate $\int_0^1 e^x dx$ by using a Reimann sum. It may be helpful to know that $\sum_{i=1}^n e^{i/n} = \frac{e^{1/n}(e-1)}{e^{1/n}-1}$

   (b) Now check your work using the Evaluation Theorem. Did you get the right answer?

8. Evaluate each of the following limits:

   (a) $\lim_{n\to\infty} \frac{\pi}{2n}[\sin(\pi/2n) + \sin(2\pi/2n) + \sin(3\pi/2n) + \cdots + \sin(\pi/2)]$

   (b) $\lim_{n\to\infty} \frac{1^5 + 2^5 + 3^5 + \cdots + n^5}{n^6}$

9. (Hard!) Recall that the formal definition of an integral is $\lim_{\max \Delta x_i \to 0} \sum_{i=1}^n f(x_i^*)\Delta x_i$ and that it is a theorem that if f is integrable, then this is the same as $\lim_{n\to\infty} \sum_{i=1}^n \frac{b-a}{n} f\left(a + \frac{b-a}{n} i\right)$. Let's see why the asumption about integrability really is necessary:

   Consider the function $f(x) = \begin{cases} 1 & x \text{ is irrational} \\ 0 & \text{otherwise} \end{cases}$

   (a) Show that using the second limit above, we would get that $\int_0^1 f(x) = 0$ and $\int_{\sqrt{2}}^{\sqrt{2}+1} f(x) = 1$

   (b) Explain why these two integrals "should be" the same.

   (c) Show that if you use the actual definition of integral (that is, the first one above), then the limit does not exist. One way to do this is to show that no matter how small a partition you choose, you can always pick the x_i^*'s so that the sum can be either 0 or 1.