Rob Bayer | Math 1A PDP Worksheet | November 13, 2008

You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Antiderivatives

1. Find all possible f:

   (a) $f'(x) = 2x - 3x^4$ (b) $f''(x) = 0$ (c) $f'(x) = \cos x + \frac{x^2}{x^2+1}$

2. If $f''(t) = 2e^t + 3\sin t$ and $f(0) = f(\pi) = 0$, what is f?

3. If a baseball is thrown straight up at 145 ft/s, when does it hit the ground?

4. A car braked with a constant deceleration of $16 ft/s^2$, and took $200 ft$ to stop. How fast was the car travelling when the brakes were first applied?

5. Prove that if F and G are both antiderivatives of f, then $F(x) = G(x) + C$ for some constant C.

Areas under curves & Definite Integrals

1. Without using any antiderivatives or riemann sums, evaluate each of the following integrals:

   (a) $\int_0^3 (\frac{1}{2}x - 1)dx$

   (b) $\int_0^2 \sqrt{4 - x^2}dx$

   (c) $\int_2^8 |x - 5|dx$

2. Express the total area of all the rectangles using sigma notation:

3. Express the following limits as areas:

   (a) $\lim_{n\to\infty} \frac{2}{n} \sum_{i=1}^{n} \left[3(1 + \frac{2i}{n}) - 6\right]$

   (b) $\lim_{n\to\infty} \frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \cdots + \sqrt{n}}{n^{3/2}}$

4. Setup, but do not try to evaluate, the Riemann sum for the area under the function $f(x) = \frac{\ln x}{x}$ for $3 \le x \le 10$

5. Find the area under $f(x) = x^2 + 2x - 5$ for $0 \le x \le 4$. DO NOT use any antiderivatives
   Hint: $1 + 2 + \cdots + n = \frac{n(n+1)}{2}$ and $1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$

6. Find the area under $f(x) = e^x$ from $x = 0$ to $x = 1$. DO NOT use any antiderivatives
   Hint: Recall that $1 + r + r^2 + r^3 + \cdots + r^n = \frac{r^{n+1}-1}{r-1}$

7. Suppose $f''(x) > 0$ on some interval I. If we use the trapezoid rule to estimate the area under f on I, do we get an over or under estimate?