You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Note: a calculator will be handy for all these problems.

Optimization Problems

1. Suppose it costs a certain toy company $c(x) = \frac{x^2}{1000} + 12x + 10$ dollars to produce x toys. If each toy sells for \$20, how many toys should the company produce in order to maximize its profits? What is $c'(x)$ at this amount? How much profit will they make?
2. Find a positive number such that the sum of the number and its reciprocal is as small as possible.
3. An oil refinery is 1km downstream and on the other side of a 100m wide river from an oil pipeline. If it costs \$10/m to build pipe along the shore and \$15/m to build pipe under water, how far downstream should the pipe run along the shore before crossing the river? What is the minimum possible cost of this project?
4. Find the dimensions of the rectangle of largest area that can be inscribed in a half-circle of radius r. You may assume one side of the rectangle lies along the straight edge of the semicircle.
5. A $10m$ long piece of wire is cut into 2 pieces. One is bent into a square and the other into an equilateral triangle. How long should each of the two pieces be so as to minimize the total area of the two shapes?
6. A 3ft by 3ft square piece of cardboard is folded into a topless box by cutting out a square from each of the corners. What is the maximum area of such a box?
7. A raingutter is to be constructed from a metal sheet of width 30cm by bending up one third of the sheet on each side to an angle θ. What should θ be to maximize the amount of water the gutter can carry?

Newton's Method

1. Let $f(x)$ be some differentiable function. At what point does the tangent line to f at the point a cross the x-axis? Does this look familiar?
2. Use Newton's method find a root of the equation $e^{-x} = 2 + x$
3. Use Newton's method to approximate $\sqrt[4]{78}$ to within 4 decimal places. Hint: recall that $\sqrt[4]{81} = 3$.
4. (a) Apply Newton's method to the the equation $\frac{1}{x} - a = 0$ to derive the following reciprocal algorithm:

$$x_{n+1} = 2x_n - ax_n^2$$

   (b) Use part (a) to compute $\frac{1}{1.6984}$ to within 6 decimal places.

   (c) Which do you think is quicker: the method outline here, or acually doing the division?