

You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

1. Consider each of the graphs pictured below. Find the intervals of increase/decrease, local extrema, inflection points, and intervals of concavity for the **original function**. If you cannot determine one of these properties from the given graph, explain why. Note that f, g, h have no relation to each other.

(a) f (b) g' (c) h''

2. Sketch a graph of each of the following functions. Be sure to explicitly label all local extrema, inflection points, intercepts, and asymptotes.

(a) $\frac{x}{x^2-9}$

(b) $x + \sqrt{|x|}$

(c) $\frac{e^x}{x}$

(d) $\tan^{-1}\left(\frac{x-1}{x+1}\right)$

3. Find the range of the function $f(x) = \frac{x^2}{x^2+3}$

4. A line $y = mx + b$ is called a *slant asymptote* of $f(x)$ if $\lim_{x \rightarrow \infty} (f(x) - (mx + b)) = 0$ or $\lim_{x \rightarrow -\infty} = 0$.

Use long division to find all slant asymptotes of $f(x) = \frac{x^2+12}{x-2}$ and use them to help you sketch a graph of f .

5. Show that $y = x + \frac{\pi}{2}$ and $y = x - \frac{\pi}{2}$ are both slant asymptotes (see above) of $y = x - \tan^{-1} x$. Use this information to help you sketch a graph of $y = x - \tan^{-1} x$

6. Suppose that f, g are both concave up everywhere. What conditions must you impose on f to ensure that $h(x) = f(g(x))$ is concave up everywhere?

7. Suppose that f, g are increasing on some interval I .

(a) Show that $f + g$ is also increasing.

(b) Show that if we also assume that f, g are positive, then fg is increasing.

(c) Show that the extra assumption in (b) really is necessary. That is, give an example of two increasing functions whose product is not increasing.

8. Suppose it costs a certain toy company $c(x) = .1x^2 + 12x + 10$ dollars to produce x toys. If each toy sells for \$20, how many toys should the company produce in order to maximize its profits? What is $c'(x)$ at this amount? How much profit will they make?