

You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

1. Use differentials to show that the relative error in finding the surface area of a sphere is approximately twice the relative error in the measurement of the radius.
2. Prove that $\sin^{-1}(\tanh x) = \tan^{-1}(\sinh x)$
3. Two people start from the same point. One walks East at 3 mi/h and the other walks Northeast at 2mi/h. How fast is the distance between them changing after 15 minutes?
4. Find the absolute maximum and minimum values of $f(x) = \sin x + \cos x$ on the interval $[0, \pi/3]$
5. Prove that if f, g are continuous on $[a, b]$, differentiable on (a, b) , $f(a) = g(a)$ and $f'(x) < g'(x)$ for all x in (a, b) , then $f(x) < g(x)$ for all x in (a, b)
6. A freshly brewed cup of coffee has temperature $95^\circ C$ in a $20^\circ C$ room. When its temperature is 70° , it is cooling at a rate of 1° per minute. At what time does this occur?
7. Evaluate each of the following
 - (a) $\lim_{x \rightarrow 0^+} x^{x^a}$, where $a > 0$
 - (b) $\lim_{x \rightarrow 1^-} (\ln x) \tanh^{-1} x$
 - (c) $\lim_{x \rightarrow 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$
8. A number a is called a fixed point of f if $f(a) = a$. Show that if $f'(x) \neq 1$ for any x , then f has at most 1 fixed point.
9.
 - (a) Find an equation of the tangent line to $y = e^x$ that is parallel to the line $x - 4y = 1$.
 - (b) Find an equation of the tangent line to $y = e^x$ that passes through the origin.
10. Show that $f(x) = e^x - x - 2$ has exactly two roots.
11. A lighthouse is on a small island 3km from the nearest point on shore. If P is that nearest point and the coastline is a straight line and the lighthouse rotates at 4 rev/min, how fast is the beam of light moving across the shore when it is 1km from P ?
12. Find the local and absolute extrema of $\frac{\ln x}{x^2}$ on the interval $[1, 3]$
13. Simplify:
 - (a) $\cos(\arctan(\sin(\operatorname{arccot} x)))$
 - (b) $\arcsin(\sin 3\pi)$
 - (c) $\log_{10} 25 + 2 \log_{10} 2$
 - (d) $e^{\sinh^{-1} 2x}$
14. Show that if f'' exists and is continuous, then $f''(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$
15. A certain function has the property that its graph passes through the point $(0, \pi)$ and that the slope of its tangent line at $(x, f(x))$ is always $2f(x)$. Find the function.