You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Hyperbolic Functions

1. Find the derivative of each of the following:

   (a) $x \cosh x$

   (b) $\sinh(\tan x)$

   (c) $e^{\tanh x}$

   (d) $x \tanh^{-1} x + \ln \sqrt{1-x^2}$

2. Find the domain of the function $f(x) = \tanh^{-1} \sqrt{x^2 - 1}$

3. Prove that $f(x) = \sinh x$ is an odd function and that $g(x) = \cosh x$ is an even function

4. If $\tanh x = \frac{4}{5}$, find $\cosh x$ and $\sinh x$.

L'Hospital and Indeterminate Forms

1. Evaluate each of the following limits. Be sure to clearly indicate when (if ever) you use L'Hospital's rule and why it is justified.

   (a) $\lim_{x \to \infty} \dfrac{e^x}{x^3}$

   (b) $\lim_{t \to 0^+} \dfrac{\ln t}{t}$

   (c) $\lim_{x \to 0} \dfrac{\sin 3x}{\sin 5x}$

   (d) $\lim_{x \to 1} \dfrac{\ln x}{\cos(\pi x) + 1}$

   (e) $\lim_{t \to 0} \dfrac{e^{2t} - 1}{t}$

   (f) $\lim_{s \to 0} \dfrac{\ln(1+s)}{se^s}$

   (g) $\lim_{x \to 0} \dfrac{x}{x+1}$

2. Compute each of the following limits. As before, be sure to clearly indicate anywhere you use L'Hospital's rule as well as it's justification.

   (a) $\lim_{x \to 0^+} \sqrt{x} \ln x$

   (b) $\lim_{x \to 0} \cos x \sin x$

   (c) $\lim_{x \to 0} (\csc x - \cot x)$

   (d) $\lim_{x \to \infty} x^{(\ln 2)/(1+\ln x)}$

   (e) $\lim_{x \to \infty} \dfrac{1}{\ln x} - e^{-x}$

   (f) $\lim_{x \to 0^+} (\tan 2x)^x$

   (g) $\lim_{x \to -\infty} x^2 e^x$

   (h) $\lim_{x \to 1} \left(\dfrac{1}{\ln x} - \dfrac{1}{x-1} \right)$

3. When discussing exponential growth, we showed that the amount of money in an account with an interest rate of r, compounded n times per year is $A(t) = A_0(1 + \frac{r}{n})^{nt}$. Use L'Hospital's rule and logarithmic limits to show that $\lim_{n \to \infty} A(t) = A_0 e^{rt}$. In other words, our formula for continuously compounded interest really does make sense.

4. In terms of m, n evaluate $\lim_{x \to 0} \dfrac{\cos mx - \cos nx}{x^2}$

5. Evaluate $\lim_{x \to \infty} (x - \ln x)$

6. Show that if f'' exists and is continuous, then $\lim_{h \to 0} \dfrac{f(x+h) - 2f(x) + f(x-h)}{h^2} = f''(x)$