You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Exponential Growth and Decay (cont)
Note: if you have calculators, feel free to use them on this problem.

1. (a) Prove that the percent change in a population with exponential growth is the same over any fixed time interval. In other words, show that the percent growth/decline in population is dependent only on the length of time you are looking at–not on the starting population, which starting year you're using, etc.

   (b) Suppose a certain population of bacteria are dying off in an exponential decay sort of way. Without using any exponentials or solving explicitly for P_0 or k, answer the following question: If there are 190,000 organisms at noon and 171,000 at 1pm, how many will there be at 2pm?

2. A thermometer is taken from a room where the temperature is $20°C$ to outside, where the temperature is $5°C$. After one minute, the thermometer reads $12°C$

   (a) What will the thermometer read 1 minute later?

   (b) When will the thermometer read $6°C$?

3. A sample of tritium-3 decays to 94.5% of its original amount after 1 year.

   (a) What is the half-life of tritium-3?

   (b) How long will it take to decay to 20% of its original amount?

4. Suppose some radioactive substance has half-life of h years, and at time $t = 0$ there is M_0 grams of it. Find a formula (depending on h, M_0) for $M(t)$, the amount of the substance after t years. Be sure to simplify your answer.

5. (The power of compound interest) Suppose you could travel back in time and invest 1 penny at 1% interest compounded continously. How much money would you have today if you made your investment:

   (a) When the U.S. was founded (1789)?

   (b) When Columbus sailed the ocean blue (1492)?

   (c) When Rome fell (476)?

   (d) In the year 0?

Inverse Trig Functions

1. Sketch the graphs of $\sin^{-1} x$ and $\frac{1}{\sin x}$ on the same axes. Are they the same?
2. Sketch the graphs of $\sin^{-1}(\sin x)$ and $\sin(\sin^{-1} x)$. Are they the same?
3. True/False. Explain the true ones, provide a counterexample for the false ones:
   (a) $\cos^{-1}(\cos x) = x$ for all x in the domain.
   (b) $\cos(\cos^{-1} x) = x$ for all x in the domain.
4. Simplify each of the following:
   (a) $\sin(\arctan \frac{1}{x})$
   (b) $\sec(\tan^{-1} 2)$
   (c) $\tan(\sin^{-1} \frac{y}{x})$
   (d) $\cos(\cos^{-1}(-\frac{1}{3}))$
   (e) $\csc(\arccos 2x)$
5. Find the derivative of each of the following:
   (a) $y = \sqrt{\tan^{-1} x}$
   (b) $y = \cos^{-1}(e^{2x})$
   (c) $y = \tan^{-1}(\ln x)$
   (d) $y = e^{\sin^{-1} 2x}$
6. Find each of the following limits:
   (a) $\lim_{x\to 0^+} \tan^{-1}(\ln x)$
   (b) $\lim_{x\to\infty} \sin^{-1}\left(\frac{x+3}{2x-4}\right)$
7. Find the domain and range of each of the following:
   (a) $\sin(\arctan(\ln x))$
   (b) $\cos^{-1} x^2$
8. Find an x such that $\cos^{-1} x = \sin^{-1} x$. You may assume such an x exists and is in the interval $[0, \pi/2]$
9. A 10 ft ladder is resting against a wall when the bottom begins to slide away from the wall at $2 ft/s$. How fast is the angle between the ladder and the wall changing when the bottom of the ladder is 6ft from the base of the wall?
10. (a) Use a right triangle to show that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for any x in the domain.
    (b) Use part (a) to prove the formula for $\frac{d}{dx} \cos^1 x$
11. Prove that the graphs of $y = \tan^{-1} x$ and $y = \cos x$ intersect.