You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Logarithmic and Exponential Derivatives

1. Find the derivative of each of the following:

   (a) $\ln(\cos\theta)$
   (b) x^2e^x
   (c) $\ln x^x$
   (d) $4^{\sin x+e^x}$
   (e) $\ln(1+\ln x)$
   (f) $2^{(3^{(t^2)})}$
   (g) e^{1/x^2}
   (h) $\ln(\sec x+\tan x)$

2. What is the domain of $f(x)=\ln(4-\sqrt{x^2-2})$?

3. (a) Use the change of base formula to find an expression for $\frac{d}{dx}\log_a x$
   (b) Use your answer to (a) to find the derivative of $\log_2(1-3x)$

4. Use logarithmic differentiation to find the derivative of each of the following:

   (a) x^x
   (b) $(\tan x)^{\sin x}$

5. Sometimes, logarithmic differentiation can be useful even for things that aren't $f(x)^{g(x)}$. Use logarithmic differentiation and log laws to find the derivatives of each of the following. Is this easier or harder than computing the derivatives directly using the power, quotient, product, etc rules?

   (a) $y=\sqrt[4]{\frac{x^2+1}{x^2-1}}$
   (b) $y=x^2e^{2x}(x^2+1)^3$

6. Find an equation of the tangent line to the curve $xe^y+ye^x=1$ at the point $(0,1)$

7. (a) Use a combination of logarithmic and implicit differentiation to find y' (in terms of y and x) for the curve $y^{\sin x}=x^{\sqrt{x}}$.
   (b) Now find y' by first explicitly solving for y and then differentiating. Do you get the same thing? Which way is easier?

8. Use logarithmic differentiation to prove that if $y=f(x)g(x)h(x)$ with $f,g,h>0$, then $y'=f'gh+fg'h+fgh'$

Exponential Growth and Decay

Note: if you have calculators, feel free to use them on this problem.

1. A thermometer is taken from a room where the temperature is $20 \deg C$ to outside, where the temperature is $5 \deg C$. After one minute, the thermometer reads $12 \deg C$
   (a) What will the thermometer read 1 minute later?
   (b) When will the thermometer read $6 \deg C$?
2. A sample of tritium-3 decays to 94.5% of its original amount after 1 year.
   (a) What is the half-life of tritium-3?
   (b) How long will it take to decay to 20% of its original amount?
3. Suppose you have a balance of 500 on a credit card with a 20% interest rate. Asuming you make no payments and there are no late fees, how much do you owe after 1 year if the interest is compounded
   (a) anually? (b) monthly? (c) daily? (d) continuously?
4. Suppose you open a savings account that advertises a 3.2% APR, compounded monthly. What is the effective Anual Percentage Yield (APY)? In other words, if you invest \$100 in this account for one year, how much interest will you earn?
5. One number of particular interest to investors and bankers is the time required for the value of an investment to double. Throughout this problem, the rate r will be given as an APY so that the value after t years is given by $A(t) = A_0(1 + \frac{r}{100})^t$
   (a) In terms of r, how long does it take an investment to double?
   (b) Many bankers/investors like to use the "rule of 72" for estimating doubling times. This rule says that the doubling time is approximately $72/r$. Using your answer to part (a), compare the actual doubling time to the "rule of 72" estimate of doubling time for $r = 2\%, 6\%, 12\%, 18\%$. Does it seem like a good rule?
   (c) Inflation can be thought of as the rate at which the cost of goods increases. If the anual inflation rate is 3.5%, how long does it take the value of \$1 to be cut in half (ie, how long until goods are twice as expensive)?