You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

**Exponentials**

1. On the same set of axes, graph $y = 1^x, y = 2^x, y = e^x, y = (1/2)^x, y = e^{-x}$, and $y = e^{(x^2)}$. Do these graphs have any points in common?

2. Find each of the following limits:

   (a) $\lim_{x\to\infty} e^{-x}$

   (b) $\lim_{x\to\infty} (1.0001)^x$

   (c) $\lim_{x\to\infty} \left(\frac{x}{1+x}\right)^x$

   (d) $\lim_{x\to-\infty} e^{x^3}\cos x$

   (e) $\lim_{x\to 2^-} e^{3/(2-x)}$

   (f) $\lim_{x\to 2^+} e^{3/(2-x)}$

3. (a) What is $\lim_{x\to\infty} a^x$ if:

   i. $a > 1$ ii. $a = 1$ iii. $0 \le a < 1$

   (b) Why do we not usually talk about the function $a^x$ for $a < 0$?

4. Find the domain of each of the following functions:

   (a) $\sqrt{1-2^t}$ (b) $\tan e^{-t^2}$ (c) $\dfrac{1}{1-e^{1-t}}$

5. Show that the curves $y = e^x$ and $y = \begin{cases} x^x & x > 0 \\ 0 & x \le 0 \end{cases}$ intersect

6. Show that there is at least one solution to $e^x = -x$

**Inverse Functions and Logarithms**

1. (a) If $f(x) = y$, what is $f^{-1}(y)$? What about $f^{-1}(x)$?
   (b) If $f^{-1}(x) = y$, what is $f(y)$? $f(x)$?
   (c) Explain why a function that is not one-to-one cannot have an inverse.

2. Determine whether each of the following functions are one-to-one. For those that are, find their inverse.

   (a) $\sin x$ (b) $\ln(x+4)$ (c) $2x^3+3$ (d) $\sqrt{x-2}$ (e) $x^2+4$

3. The function $f(x) = \frac{4x-1}{2x+3}$ is one-to-one, though it's rather difficult to show.

   (a) Find $\left(f^{-1}\right)'(x)$ using the derivative formula for inverse functions.
   (b) Now find $\left(f^{-1}\right)'(x)$ by first finding $f^{-1}(x)$. Do you get the same thing?

4. Suppose $f$ is one-to-one. How are the domain and range of $f$ related to the domain and range of $f^{-1}$?

5. Simplify each of the following.

   (a) $\log_5 10 + \log_5 20 - 3\log_5 2$
   (b) $\ln e^{\sqrt{x}} + e^{x\ln x}$
   (c) $\ln(x^2-4) - \ln(x+4)$
   (d) $\log_8 2$

6. Solve for $x$:

   (a) $\log_2(x+3) = 2$ (b) $e^{2x+3} - 6e^x = 0$ (c) $\log_3(x+1) + \log_3(x-1) = 1$

7. Let $f(x) = \ln(2 + \ln x)$

   (a) Find the domain of $f$

(b) What is $f^{-1}(x)$? What is its domain?

8. Find each of the following limits:

   (a) $\lim_{x \to 2^{-1}} \ln(2 - x)$

   (b) $\lim_{x \to 2} \ln(2 - x)$

   (c) $\lim_{x \to 0^+} \ln \sin x$

   (d) $\lim_{x \to \infty} \ln(1 + x^2) - \ln(1 - x^2)$

9. Prove the change of base formula: $\log_b x = \frac{\ln x}{\ln b}$

10. Without using a calculator, determine which is larger: $\log_{10} 99$ or $\log_9 82$

11. Prove, using the $\epsilon - N$ definition of a limit, that $\lim_{x \to -\infty} e^x = 0$