You should work on the following problems in groups of 3. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Linear Approximations

1. Find the linearization of each of the following functions at the given points:
   (a) $x^{3/4}$, at $a = 16$
   (b) $\sqrt{x}$, at $a = 25$
   (c) $\sin x$, at $a = \pi/2$
2. Use a linearization to estimate each of the following. Compare your answer to the actual value using a calculator. By sketching a graph, explain why it makes sense geometrically that your linearization is an under/over estimate.
   (a) $\sqrt{99.7}$
   (b) $\tan(-0.2)$
   (c) $\frac{1}{1002}$
   (d) $\sec(0.1)$
3. In order to simplify their calculations, physicsists love to use the approximation $\sin\theta \approx \theta$ for θ near 0. Using linearization, explain why this makes sense.
4. (Why Tangents?) Let's say we knew nothing about linearization or tangent lines and just wanted to approximate our function $f(x)$ by some linear function $p(x) := A(x - a) + B$ and make it "look like" $f(x)$ near the point a.
   (a) Clearly we want $f(a) = p(a)$. What does this mean we should pick for A and B?
   (b) After a little thought, we decide that since we want $p(x)$ to "look like" $f(x)$ near a, we should also want $p'(a) = f'(a)$. What does this tell you about A and B? Does this formula look familiar?
5. (To Quadratic, and Beyond!) After working through the previous problem, you may (or may not) be asking yourself "why stop at linear?" And the answer that there is no reason to. In particular, let's say we want to approximate $f(x)$ by some $p(x) := a_0 + a_1(x - a) + a_2(x - a)^2$.
   (a) If we want $f(a) = p(a), f'(a) = p'(a), f''(a) = p''(a)$, how should we pick a_0, a_1, a_2?
   (b) What if we want p to be cubic? Quartic?
   (c) In general, what should the a_n's be so that $p(x) = a_0 + a_1(x - a) + a_2(x - a)^2 + a_3(x - a)^3 + \cdots$ is a good approximation to $f(x)$ near a?[1]

Differentials

1. (a) Sketch two concentric circles, one of radius r and one of radius $r + \Delta r$. Shade in the area representing ΔA
   (b) Using the fact that $A = \pi r^2$, find expressions for dA and ΔA
   (c) Why is dA a good approximation to ΔA?
   (Hint: recall that the inner circle in your drawing has circumference $2\pi r$)
   (d) In general, what, if anything, is the difference between dx and Δx? Between dy and Δy? When is $dy \approx \Delta y$?
2. Let $y = \sqrt{x}$.
   (a) Find a formula for dy.
   (b) Find dy and Δy if $x = 16$ and $dx = 1$. Which is easier to calculate?
   (c) Draw a graph labeling $y, dy, \Delta y$, and dx for the situation described above.
3. Find the differential of each of the following:
   (a) $\frac{1}{x^2+2}$
   (b) $\sqrt{3 + \sin x}$

[1] If you've taken AP Calc BC, this formula should look familiar. If not, then you'll see them (in the form of Taylor series) in 1B

4. The radius of a circular disk is given as 24cm, with a maximum error in measurement of 0.5cm
   (a) Use differentials to estimate the possible error in the calculated area of the disk.
   (b) What is the relative error?
   (c) What is the **actual** maximum possible error in the calculated area of the disk?
5. (a) Prove that the relative error in the volume of a sphere is approximately 3 times the relative error of the radius.
   (b) What is the relation between the relative error of surface area and that of the radius?
   (c) Combining your answers to parts (a) and (b), how is the relative error of volume related to the relative error of surface area?
6. Estimate the number of cubic cm's of paint needed to apply a coat of paint 0.05cm thick to a hemispherical dome with diameter $50m$.
7. Use differentials to estimate the value of $\sec 0.08 - 1$ (Hint: $\sec 0 = 1$, so I'm asking for an estimate of the error in estimating $\sec x$ near $x = 0$)