

**Related Rates**

- Suppose you are on top of a 10ft ladder that is leaning against a wall, with the base of the ladder 3ft away from the wall. If your nemesis kicks out the bottom of the ladder so that it travels at a constant 2ft/s, how fast are you falling when you are 1 ft off the ground? How about after 2 seconds?
- While working as a boat valet for a restaurant on one of the local lakes<sup>1</sup> one of your jobs is to pull customers' boats into the dock using a rope tied to the front of their boat. If you hold the rope 1m above the edge of the dock, and take in rope at a rate of 1m/s, how fast is the boat approaching the dock when it is 8m away?
- In Search and Rescue operations for missing persons, one of the most important things is establishing a Search Area, which is a circle centered at the last known location of the person and radius equal to the maximum distance they could have travelled in the time they've been missing<sup>2</sup>. Suppose you're looking for a hiker who can walk at a maximum of 4ft/s. At what rate is the area of your Search Area increasing after the person has been missing for 1 hour? For 2 hours? For 1 day?
- While working for a party supply store, your job is to fill balloons from a helium tank. If the helium tank can output gas at  $.5 \text{ ft}^3/\text{s}$ , at what rate is the volume of a spherical balloon increasing after 5 seconds? At what rate is its surface area changing when the volume is  $2 \text{ ft}^3$ ?
- A slide at the new playground is shaped like the hyperbola  $y = 1/x$ . If a kid's vertical speed is a constant  $1 \text{ m/s}$ , what is his horizontal speed when  $x = 2$ ?
- Radar guns (as used by baseball, police officers, etc) work by calculating the rate of change of the distance between the gun and the object being tracked. Since the object may not be coming straight at the radar gun, the speed of the object and the speed registered by the gun may be different. Let's explore:
  - A police officer is sitting at the side of the road, pointing his radar gun at the far lane of traffic, which is 20ft away. Now suppose you are driving your car down the road at 150 ft/s (approx 100 mi/h). What will the cop's radar gun read when you are 500 ft down the road? 100 ft? 10 ft? 0 ft?
  - Is it possible for the radar gun to ever register a speed faster than what you are actually driving? Why or why not?
  - Now suppose you're the cop and your radar gun reads 90ft/s for a car 500 ft down the road (again, still in that far lane which is 20ft away)? If the speed limit is 100ft/s (approx. 70 mi/h), can you give this driver a ticket? What if the car is only 50 ft away? 10ft?
- Suppose you are 5ft tall and are a photographer taking pictures of the International Hot Air Balloon Festival which is held each year in Albuquerque, NM<sup>3</sup> From a position 200 ft away from the takeoff point, your favorite balloon starts rising at a constant rate of 10 ft/s.
  - If you track the balloon the whole way up, at what rate must you be changing the angle between your camera and the ground after 5 seconds?
  - In order to keep the balloon in focus, you also need to know at what rate the distance between you and the balloon is changing. What is this rate after 5 seconds?
- Suppose you are a stage actor, and there is a bright light mounted on the ground at the edge of the stage pointed at the curtain which is 50 ft from the light. If you are 5 ft tall and walk at 5ft/s in a straight line from the light to the curtain, at what rate is the height of your shadow changing when you are 10 ft from the curtain? At what rate is the angle from the light to the top of your shadow changing?
- At noon, ship A is 150km west of ship B. Ship A is sailing east at 35km/h and ship B is sailing north at 25km/h. How fast is the distance between the ships changing at 4pm?
- On a hot August day you order a snowcone, but sadly are unable to eat it before it melts. By some great cosmic coincidence, the liquid left over from melted ice is exactly enough to fill the paper cone part, which is 3in tall and has a radius of 1in. Using a straw, you decide to sip out the sugary colored water a rate of  $.5 \text{ in}^3/\text{s}$ . How fast is the height of the liquid in the cone decreasing after 5 seconds? Recall that the volume of a cone is  $\frac{1}{3}\pi r^2 h$
- A lighthouse is on a small island 3km from the nearest point on shore. If  $P$  is that nearest point and the coastline is a straight line and the lighthouse rotates at 4 rev/min, how fast is the beam of light moving across the shore when it is 1km from  $P$ ?
- The minute hand on a watch is 8mm long and the hour hand is 4mm long. How fast is the distance between the tips of the two hands changing at 1 o'clock?

<sup>1</sup>No, I'm not making this up. We Minnesotans really do do this.

<sup>2</sup>In truth, the situation is a little more complicated. Natural features (rivers, cliffs, roving barbarian hordes) can all narrow it down

<sup>3</sup>But seriously. If you ever get the chance, you should go—it's a jolly good time