Name: _____

Math 1A Quiz 6
October 14, 2008
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You have until 4:30 to complete this quiz. You must show your work.

1. (3 pts) Find an explicit formula for $f^{-1}(x)$ if $f(x) = \frac{1+e^x}{1-e^x}$. You may assume without proof that f is 1-1.

$$y = \frac{1+e^x}{1-e^x}$$

$$y = 1 + (1+y)e^x$$

$$\frac{y-1}{y+1} = e^x$$

$$f^{-1}(x) = \ln\left(\frac{x-1}{x+1}\right)$$

2. (3 pts) Find y' in terms of y and x if $y^{3x} = (\sin x)^{(e^{2x})}$.

$$3x \ln y = e^{2x} \ln(\sin x)$$

$$3\ln y + \frac{3x}{y} y' = 2e^{2x} \ln(\sin x) + \frac{e^{2x}}{\sin x} \cos x$$

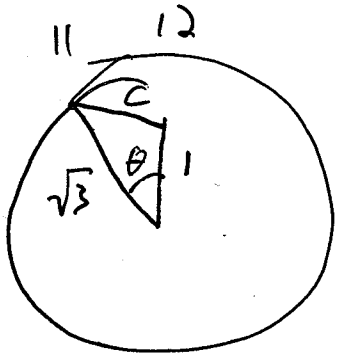
$$y' = \frac{\left(2e^{2x} \ln(\sin x) + \frac{e^{2x}}{\sin x} \cos x - 3\ln y\right) y}{3x}$$

3. (4 pts) In order to generate the 1.21 jigowatts necessary to send Oski back to the future, Doc Hald is busy climbing the Campanille to attach a lightning rod to capture the bolt of lightning that will be coming at midnight.

Through a series of freak accidents, at 11:55pm he ends up standing on the very tip of the minute hand, needing to jump to the very tip of the hour hand. If the minute hand is $\sqrt{3}$ meters long and the hour hand is $1m$ long, how fast is the distance between the tip of the minute hand and the tip of the hour hand changing? In order to simplify the problem, you may assume that the hour hand is pointing straight at the 12 and is not moving.

Hint 1: $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}, \sin\frac{\pi}{6} = \frac{1}{2}$; $\cos\frac{\pi}{3} = \frac{1}{2}, \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$

Hint 2: What is the angle between a hand pointing at the 11 and a hand pointing at the 12?



$c^2 = 1^2 + \sqrt{3}^2 - 2 \cdot 1 \cdot \sqrt{3} \cos\theta$

want: $\frac{dc}{dt}$ when $\theta = \frac{1}{12} \cdot 2\pi = \frac{\pi}{6}$.

when $\theta = \frac{\pi}{6}$: $c^2 = 1 + 3 - 2\sqrt{3} \cdot \frac{\sqrt{3}}{2} = 1$

so $c = 1$

$2c\frac{dc}{dt} = 2\sqrt{3} \cdot \sin\theta \cdot \frac{d\theta}{dt}$.

$\frac{d\theta}{dt} = 2\pi$ rad/h since it measures minute hand

@ $c=1, \theta=\frac{\pi}{6}$: $\not{2}\frac{dc}{dt} = \not{2}\sqrt{3} \cdot \frac{1}{\not{2}} \cdot 2\pi = \sqrt{3}\pi$ m/h