Name: ______________________________

Math 1A Quiz 3

September 16, 2008

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You have until 4:30 to complete this quiz. You must show your work.

1. (2 pts) Complete the following definitions:

   (a) $\lim_{x \to a^+} f(x) = L$

   $$\forall \varepsilon > 0 \exists \delta > 0 : 0 < x - a < \delta \Rightarrow |f(x) - L| < \varepsilon$$

   (b) $\lim_{x \to a} f(x) = \infty$

   $$\forall M \exists \delta > 0 : 0 < |x - a| < \delta \Rightarrow f(x) > M$$

   (c) $\lim_{x \to -\infty} f(x) = L$

   $$\forall \varepsilon > 0 \exists M : x < M \Rightarrow |f(x) - L| < \varepsilon$$

   (d) $\lim_{x \to \infty} f(x) = -\infty$

   $$\forall M \exists N : x > N \Rightarrow f(x) < M$$

2. (3 pts) Prove that the equation $\sqrt[3]{x} = 1 - x$ has at least one solution.

   Let $f(x) = \sqrt[3]{x} + x - 1$. Note that $f(x)$ is continuous everywhere so we can use the IVT. Also, any solution to $f(x) = 0$ will be a solution to the original equation.

   $f(-1) = -1 - 1 - 1 = -3$

   $f(1) = 1 + 1 - 1 = 1$

   Since $f(-1) < 0 < f(1)$, by the IVT, there is a c between -1 and 1 such that $f(c) = 0$, as was to be shown.

3. (3 pts) Find $\lim_{x \to \infty} \sqrt{x^2 + x} - x$

$$\begin{aligned}
\lim_{x \to \infty} \sqrt{x^2 + x} - x &= \lim \frac{(\sqrt{x^2 + x} - x)(\sqrt{x^2 + x} + x)}{\sqrt{x^2 + x} + x} \\
&= \lim \frac{x^2 + x - x^2}{\sqrt{x^2 + x} + x} \\
&= \lim \frac{x}{x(\sqrt{1 + \frac{1}{x}} + 1)} \\
&= \frac{1}{\sqrt{1 + 0} + 1} \\
&= \frac{1}{2}
\end{aligned}$$

4. (2 pts) Using the definition of derivative, find $f'(2)$ where $f(x) = 3 - 2x + 4x^2$. Note: if you use differentiation rules, you will receive **NO CREDIT** for this problem.

$$
\begin{aligned}
f'(2) &= \lim_{x\to 2} \frac{3-2x+4x^2-(3-2(2)+4(2)^2)}{x-2} \\
&= \lim \frac{4x^2-2x-12}{x-2} \\
&= \lim \frac{2(x-2)(2x+3)}{x-2} \\
&= 2(2(2)+3) = 14
\end{aligned}
$$